

Nucleon Form Factors in time-like and space-like regions


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Probing transverse nucleon structure at high momentum transfer
ECT* - European Center for Theoretical Studies in Nuclear Physics and Related Areas
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AGENDA

Nucleon Electromagnetic Form Factors

-  Definition and properties


The space-like region

-  Proton radius

-  Rosenbluth versus Akhiezer-Rekalo

The time-like region

-  Unphysical region

-  Threshold

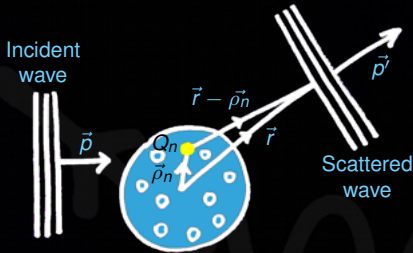
-  An amazing effect

The asymptotic region

Analytic $R = \mu G_E / G_M$

Conclusions

SEMI-CLASSICAL DEFINITION



Amplitude of the scattered wave

$$\mathcal{A}_n(\vec{r}) = Q_0 e^{i \vec{p}' \cdot \vec{r}} Q_n e^{i (\vec{p} - \vec{p}') \cdot \vec{\rho}_n}$$

$$\mathcal{A}(\vec{r}) = \sum_n \mathcal{A}_n(\vec{r}) = Q_0 e^{i \vec{p}' \cdot \vec{r}} \sum_n Q_n e^{i \vec{q} \cdot \vec{\rho}_n}$$

Form factor

$$F(\vec{q}) = \frac{1}{Q_0} \langle \phi | \sum_n Q_n e^{i \vec{q} \cdot \vec{\rho}_n} | \phi \rangle$$

The **unpolarized** differential cross section on an extended object

$$\frac{d\sigma}{d\Omega} = |F(\vec{q})|^2 \left(\frac{d\sigma}{d\Omega} \right)_{\text{pointlike}}$$

Form factor **modulus** is obtained by comparing theory and experiment

In the Breit frame the Fourier transform of the form factor

$$\rho(\vec{r}) = \frac{Q_0}{(2\pi)^3} \int d^3 \vec{q} F(\vec{q}) e^{-i \vec{q} \cdot \vec{r}}$$

is the **charge spatial distribution**.

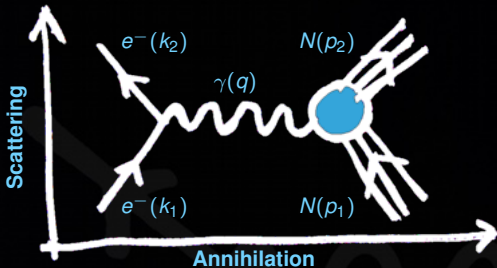
The form factor is Fourier transform of $\rho(\vec{r})$

$$F(\vec{q}) = \frac{1}{Q_0} \int d^3 \vec{r} \rho(\vec{r}) e^{i \vec{q} \cdot \vec{r}}$$

NUCLEON ELECTROMAGNETIC FORM FACTORS

- Form factors characterize the internal structure of a hadron
 $\Rightarrow F_{\text{point-like}} = \text{constant}$.
- Elastic form factors contain information on the hadron ground state.
- In a parity and T -invariant theory, the electromagnetic structure of a particle of spin $S\hbar$ is defined by **$2S + 1$ form factors**.
- Neutron and proton form factors are different.
- Playground for **theory** and **experiment**:
 - at low q^2 probe the size of the hadron;
 - at high q^2 test the QCD counting rule.

DIRAC AND PAULI FORM FACTORS



- Scattering: $e^- N \rightarrow e^- N$
Space-like kinematic region

$$q^2 = -2\omega_1\omega_2(1 - \cos\theta_e) \leq 0$$

- Annihilation: $e^+ e^- \leftrightarrow N\bar{N}$
Time-like kinematic region

$$q^2 = 4\omega^2 > 0$$

Scattering amplitude
in **Born** approximation

$$\mathcal{M} = \frac{1}{q^2} [e \bar{u}(k_2) \gamma_\mu u(k_1)] \underbrace{[e \bar{U}(p_2) \Gamma^\mu(p_1, p_2) U(p_1)]}_{\text{Nucleon EM 4-current: } J_N^\mu}$$

From Lorenz and gauge invariance

$$\Gamma^\mu(p_1, p_2) = \gamma^\mu F_1^N(q^2) + \frac{i\sigma^{\mu\nu} q_\nu}{2M_N} F_2^N(q^2)$$

- Dirac FF: $F_1^N(q^2)$, $F_1^N(0) = Q_N$

- Pauli FF: $F_2^N(q^2)$, $F_2^N(0) = \kappa_N$

$Q_N = N$ electric charge

$\kappa_N = N$ anomalous magnetic moment

SACHS FORM FACTORS

Breit frame

No energy exchanged

$$p_1 = (E, -\vec{q}/2)$$

$$p_2 = (E, \vec{q}/2)$$

$$q = (0, \vec{q})$$

Nucleon electromagnetic four-current

$$J_N^\mu = (J_N^0, \vec{J}_N) \quad \left\{ \begin{array}{l} \rho_q = J_N^0 = e \left[F_1^N + \frac{q^2}{4M_N^2} F_2^N \right] \\ \vec{J}_N = e \bar{U}(p_2) \vec{\gamma} U(p_1) \left[F_1^N + F_2^N \right] \end{array} \right.$$

Sachs Nucleon Form Factors

$$G_M^N(q^2) = F_1^N(q^2) + F_2^N(q^2)$$

$$G_E^N(q^2) = F_1^N(q^2) + \frac{q^2}{4M_N^2} F_2^N(q^2)$$

In the Breit frame represent the **Fourier transforms** of **charge** and **magnetic moment spatial distributions** of the nucleon

Normalization at $q^2 = 0$

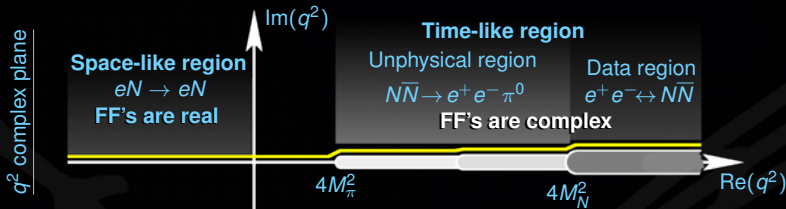
$$\text{⬢ } G_E^N(0) = \mathcal{Q}_N$$

$$\text{⬢ } G_M^N(0) = \mu_N$$

$$\mu_N = \mathcal{Q}_N + \kappa_N$$

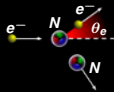
is the nucleon magnetic moment

CROSS SECTIONS AND ANALYTICITY



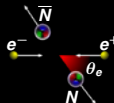
$$\text{Crossing: tot. helicity} = \begin{cases} 1 \Rightarrow G_E \\ 0 \Rightarrow G_M \end{cases}$$

$$G_E(4M_N^2) = G_M(4M_N^2)$$



Elastic scattering

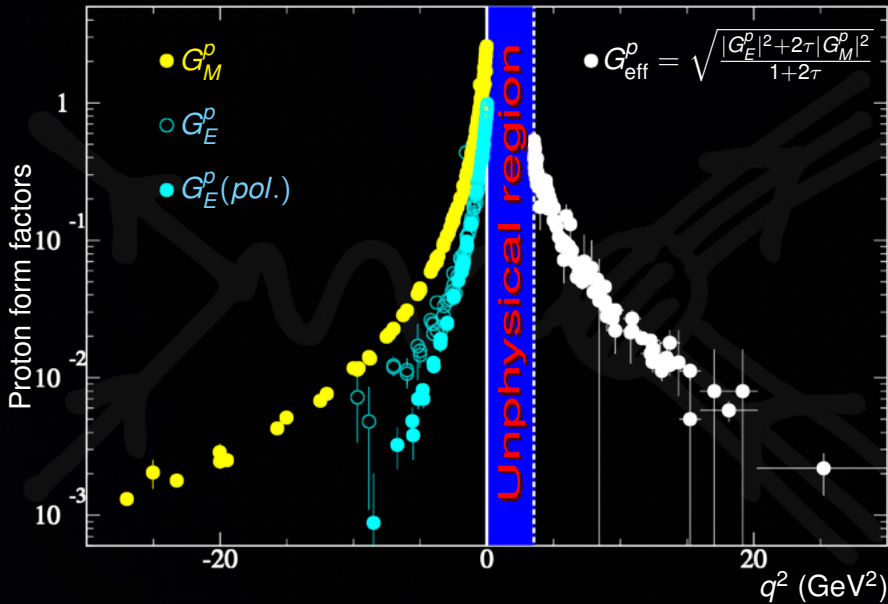
$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2 \omega_2 \cos^2 \frac{\theta_e}{2}}{4\omega_1^3 \sin^4 \frac{\theta_e}{2}} \left[G_E^2 - \tau \left(1 + 2(1-\tau) \tan^2 \frac{\theta_e}{2} \right) G_M^2 \right] \frac{1}{1-\tau} \quad \tau = \frac{q^2}{4M_N^2}$$



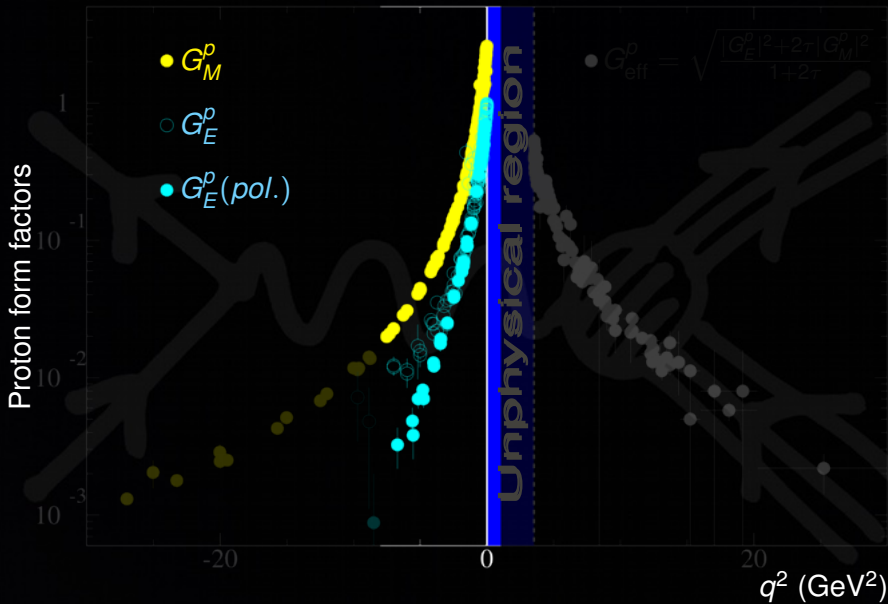
Annihilation

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2 \beta C}{4q^2} \left[(1 + \cos^2 \theta) |G_M|^2 + \frac{1}{\tau} \sin^2 \theta |G_E|^2 \right] \quad \beta = \sqrt{1 - \frac{1}{\tau}}$$

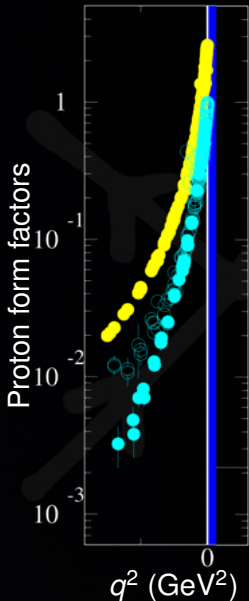
THE PROTON RADIUS



THE PROTON RADIUS



THE PROTON RADIUS



$$G_E^p(q^2) = \int d^3\vec{r} \rho(r) e^{i\vec{q}\cdot\vec{r}} = 1 + \frac{1}{6} q^2 \langle r_c^2 \rangle + \mathcal{O}(q^4)$$

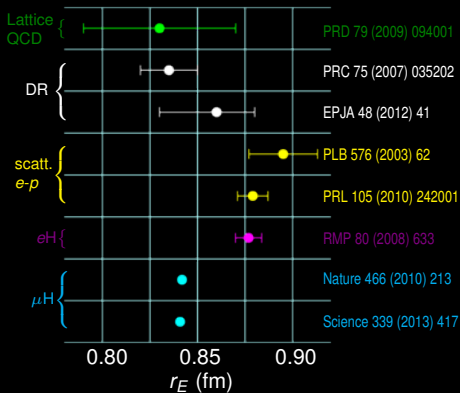
$\rho(r)$: normalized spherical charge density

The charge radius

$$r_E = \sqrt{\langle r_c^2 \rangle} = \sqrt{4\pi \int_0^\infty r^4 \rho(r) dr} = \sqrt{\frac{6}{G_E^p(0)} \left. \frac{dG_E^p}{dq^2} \right|_{q^2=0}}$$


Charge density $\rho(r)$	Form factor $G_E^p(q^2)$	Charge radius r_E	Comments
$\delta^3(r)$	1	0	pointlike
$e^{-\lambda r}$	$\lambda^4 / (q^2 + \lambda^2)^2$	$2\sqrt{3}/\lambda$	dipole
$e^{-\lambda r}/r$	$\lambda^2 / (q^2 + \lambda^2)$	$\sqrt{6}/\lambda$	monopole
$e^{-\lambda r^2}/r^2$	$e^{-q^2/(4\lambda^2)}$	$1/\sqrt{2\lambda}$	gaussian


THE PROTON RADIUS





Analyticity via dispersion relations and QCD counting rules can give directly the proton radius. . .

Ongoing discussions. . .

 $q^2 \rightarrow 0^-$ extrapolation

 Radiative corrections

 Two-photon exchange

 Coulomb corrections

Logarithmic derivative of form factor at $q^2 = 0$ by means of dispersion relations for the logarithm

$$r_E^2 = \frac{12M_\pi^2}{\pi} \int_{4M_\pi^2}^{\infty} \frac{\ln |G_E^p(t)/G_E^p(0)|}{t^2 \sqrt{t - 4M_\pi^2}} dt$$

UNPOLARIZED CROSS SECTION POLARIZATION OBSERVABLES

N.M.Rosenbluth, Phys. Rev. 79, 615

A.I.Akhiezer, M.P.Rekalo, Sov. Phys. Dokl. 13, 572

Rosenbluth formula

$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega} \right)_{\text{Mott}} \frac{1}{1-\tau} \left[G_E^2 - \frac{\tau}{\epsilon} G_M^2 \right] \quad \tau = \frac{q^2}{4M_N^2}$$



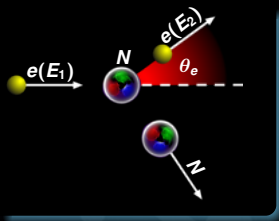
Mott pointlike cross section



Degree of linear polarization of the virtual photon

$$\left(\frac{d\sigma}{d\Omega} \right)_{\text{Mott}} = \frac{4\alpha^2}{(-q^2)^2} \frac{E_2^3}{E_1} \cos^2(\theta_e/2)$$

$$\epsilon = \left[1 + 2(1-\tau) \tan^2(\theta_e/2) \right]^{-1}$$



In case of **polarized electrons** ($h = \pm 1$) on unpolarized nucleon target and measuring the polarization of the outgoing proton:

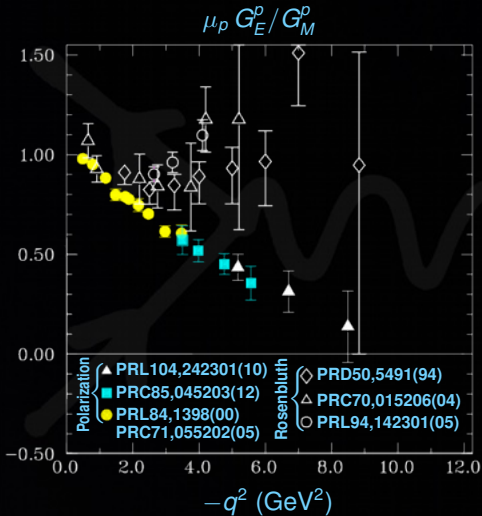
$$P'_x = -\frac{2\sqrt{\tau(\tau-1)}}{G_E^2 - \frac{\tau}{\epsilon} G_M^2} G_E G_M \tan\left(\frac{\theta_e}{2}\right)$$

$$P'_z = \frac{(E_e + E'_e) \sqrt{\tau(\tau-1)}}{M(G_E^2 - \frac{\tau}{\epsilon} G_M^2)} G_M \tan^2\left(\frac{\theta_e}{2}\right)$$



$$\frac{P'_x}{P'_z} = -\frac{2M \cot(\theta_e/2)}{E_e + E'_e} \frac{G_E}{G_M}$$

$\mu_p G_E^p / G_M^p$: ROSENBLUTH AND POLARIZATION TECHNIQUES



“Standard” dipole for the proton magnetic form factors G_M^p



Linear deviation from the dipole for the electric proton form factor G_E^p



QCD scaling still not reached



Zero crossing for G_E^p

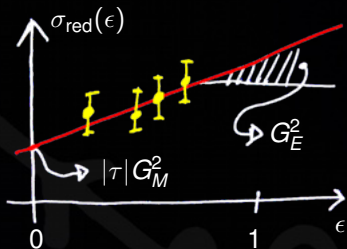


Polarization data do not agree with old Rosenbluth data (◇)



New Rosenbluth separation data from JLab **still do not agree** with polarization data

THE ROSENBLUTH METHOD

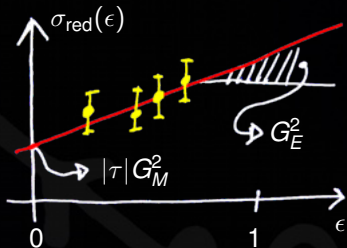


$$\text{Reduced cross section } \frac{d\sigma/d\Omega}{(d\sigma/d\Omega)_{\text{Mott}}} \epsilon(1 - \tau)$$

$$\sigma_{\text{red}}(\epsilon; \theta, Q^2) = \epsilon G_E^2 + |\tau| G_M^2$$

- Function of ϵ at fixed θ and Q^2
- Slope $\rightarrow G_E^2$
- Intercept $\rightarrow G_M^2$

THE ROSENBLUTH METHOD



$$\text{Reduced cross section } \frac{d\sigma/d\Omega}{(d\sigma/d\Omega)_{\text{Mott}}} \epsilon(1 - \tau)$$

$$\sigma_{\text{red}}(\epsilon; \theta, Q^2) = \epsilon G_E^2 + |\tau| G_M^2$$

- Function of ϵ at fixed θ and Q^2
- Slope $\rightarrow G_E^2$
- Intercept $\rightarrow G_M^2$

$$\sigma_{\text{red}}(\epsilon) = \underbrace{\epsilon G_E^2}_{<3\%} + \underbrace{|\tau| G_M^2}_{>97\%}$$

$$\delta \sigma_{\text{red}}(\epsilon) \sim 1.5\%$$

For $Q^2 \geq 4 \text{ GeV}^2$ the G_E^2 contribution to the reduced cross section can be of the same order of the error

\Rightarrow Large correlation

\Rightarrow Large errors

\Rightarrow Large rad. corr.

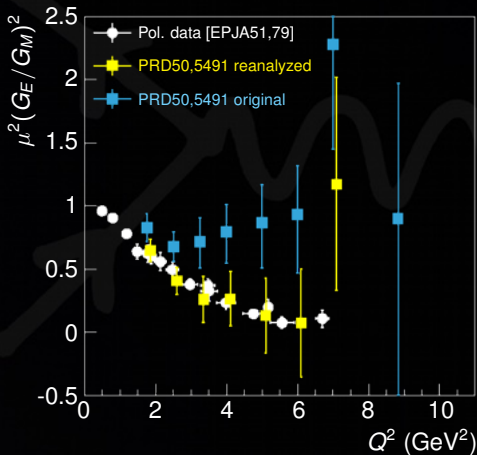
THE ROSENBLUTH RE-ANALYSIS₁

ARXIV:1604.02421

"New" fit function

$$\sigma_{red}(\epsilon) = G_M^2 (R^2 \epsilon + |\tau|)$$

- It reduces the effect of the corrections on the individual form factors
- The general normalization and systematic errors absorbed by G_M^2

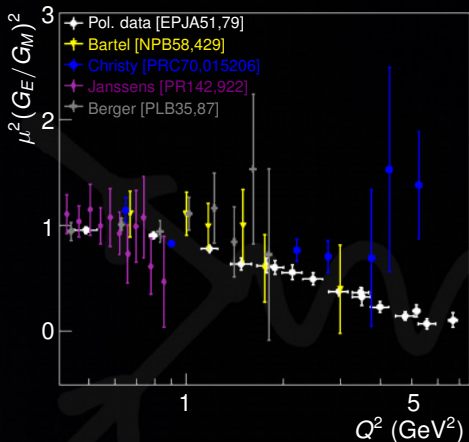


$$R^2 = \frac{G_E^2}{G_M^2}$$

Reference data from PRD50,5491

Good agreement with pol. data

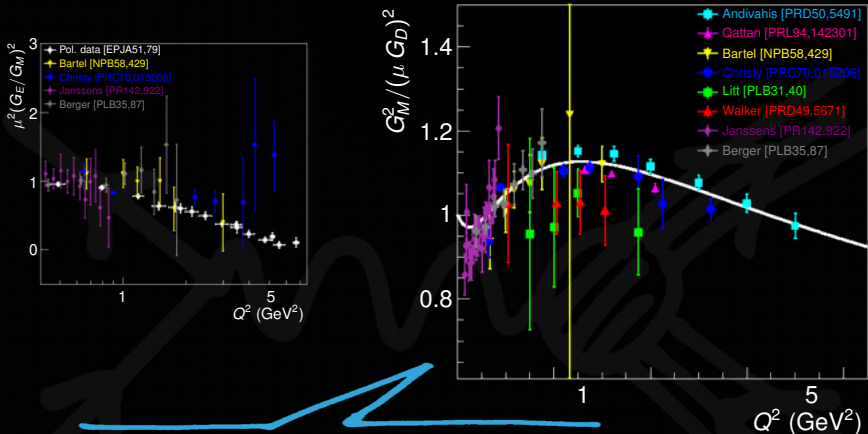
No other reaction mechanisms
(two-photon exchange) are needed



Other sets of data agree with polarization measurements

Those not in agreement, Qattan [PRL94, 142301] and Walker [RD49, 5671], show values of R^2 growing with Q^2

For these experiments radiative corrections and/or correlations are especially large



The parameter G_M^2 is better determined than R^2 by the Rosenbluth fit

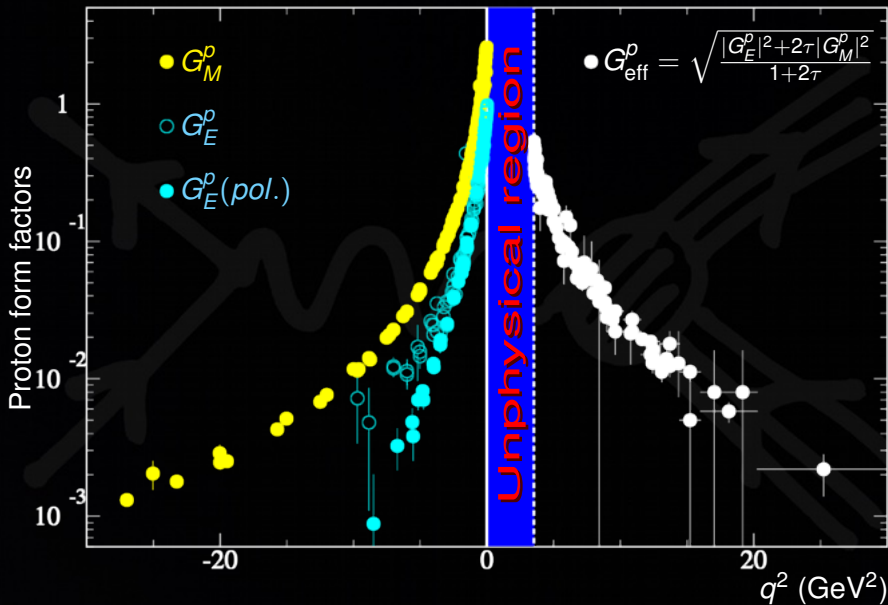


Its values are well described by two component nucleon model based on VMD [PRC69, 068201]

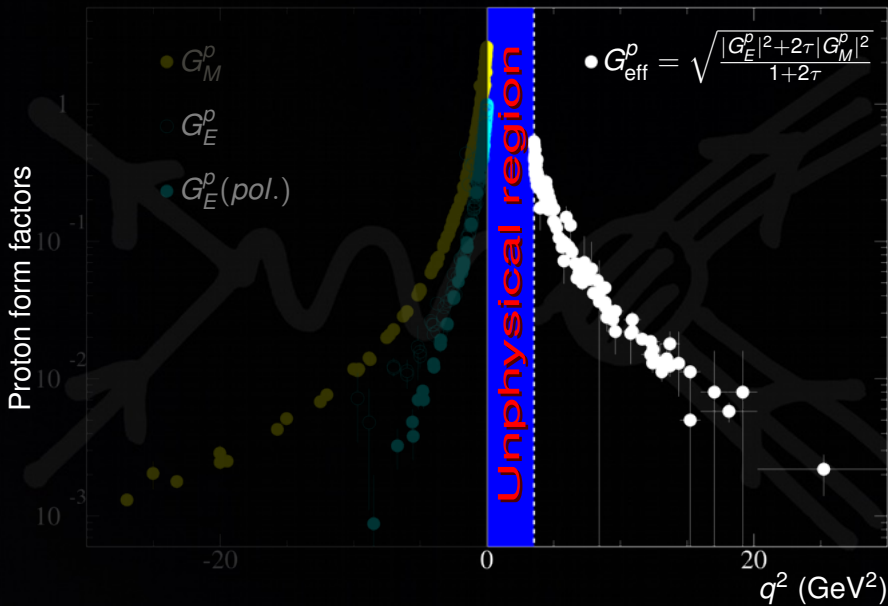


Values consistent with the model that represents, in fact, a global fit to the data

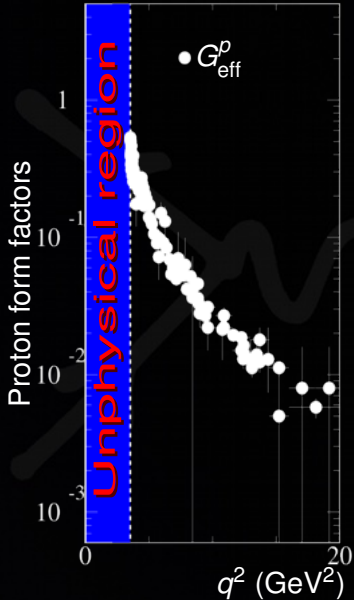
THE TIME-LIKE REGION



THE TIME-LIKE REGION



THE TIME-LIKE REGION



Differential cross section $e^+e^- \rightarrow p\bar{p}$

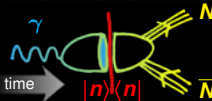
A. Zichichi, S. M. Berman, N. Cabibbo, R. Gatto [NC XXIV (1962) 170]

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2 \beta C}{4q^2} \left[(1 + \cos^2 \theta) |G_M^p|^2 + \frac{1}{\tau} \sin^2 \theta |G_E^p|^2 \right]$$





Optical theorem

$$\text{Im} \langle \bar{N}(p') N(p) | j^\mu | 0 \rangle \sim \sum_n \langle \bar{N}(p') N(p) | j^\mu | n \rangle \langle n | j^\mu | 0 \rangle$$

$|n\rangle$ are on-shell intermediate states: $2\pi, 3\pi, 4\pi, \dots$



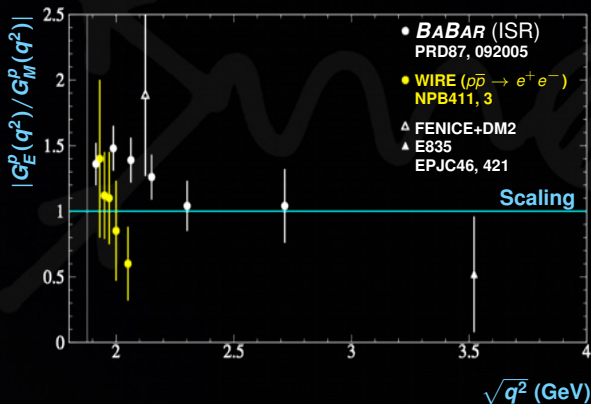
Form factors are complex for $q^2 > 4M_\pi^2$

-  The cross section is an **even function of $\cos \theta$**
-  It does **not depend on the form factor phases**
-  At high q^2 the $|G_E^p|^2$ contribution is suppressed
-  The **unphysical region is not accessible** through the annihilations $e^+e^- \leftrightarrow p\bar{p}$

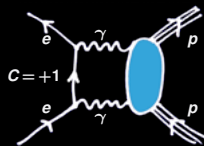
TIME-LIKE $|G_E^p/G_M^p|$ MEASUREMENTS

See next talks by
Vladimir Druzhinin
Cristina Morales
Iris Zimmermann

$$\frac{d\sigma}{d\cos\theta} = \frac{\pi\alpha^2\beta C}{2q^2} |G_M^p|^2 \left[(1 + \cos^2\theta) + \frac{4M_p^2}{q^2} \sin^2\theta \left| \frac{G_E^p}{G_M^p} \right|^2 \right]$$



$\gamma\gamma$ exchange



$\gamma\gamma$ exchange interferes with the Born term

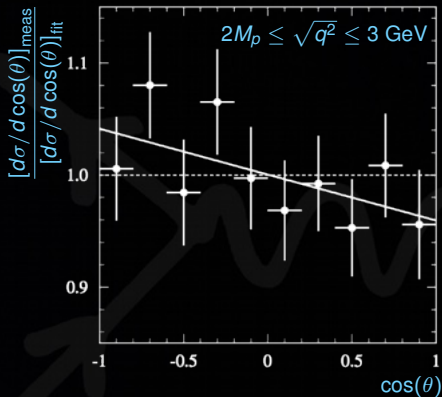


Asymmetry in angular distributions

[E. Tomasi-Gustafsson,
G.I. Gakh, NPA771,169(06)]

$\gamma\gamma$ EXCHANGE FROM $e^+e^- \rightarrow p\bar{p}\gamma$ BABAR 2013 DATA

E. Tomasi-Gustafsson, E.A. Kuraev, S. Bakmaev, SP, Phys. Lett. B659 (2008) 197
BABAR Phys. Rev. D87 (2013) 092005



See next talk by
Vladimir Druzhinin

Integrated over the $p\bar{p}$ -CM energy
from threshold up to 3 GeV

The MC-fit assumes
one-photon exchange

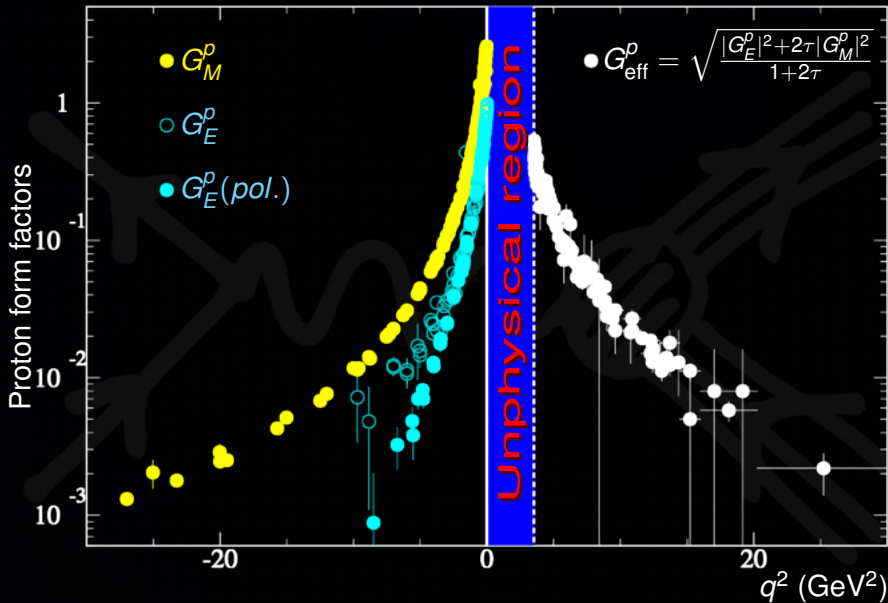
Slope = $-0.041 \pm 0.026 \pm 0.005$

Integral asymmetry

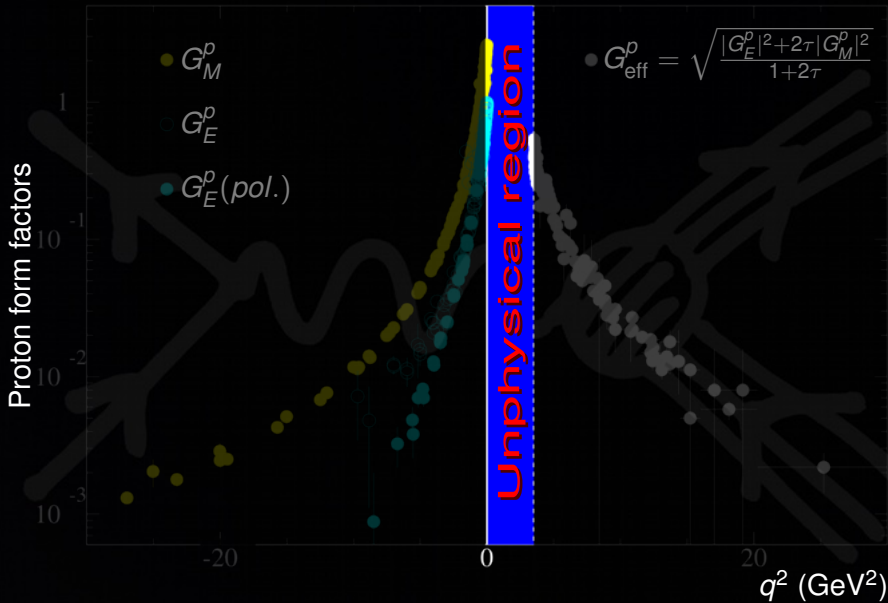
$$\langle \mathcal{A} \rangle_{\cos \theta_p} = \frac{\sigma(\cos \theta_p > 0) - \sigma(\cos \theta_p < 0)}{\sigma(\cos \theta_p > 0) + \sigma(\cos \theta_p < 0)} = -0.025 \pm 0.014 \pm 0.003$$

$\sigma(\cos \theta_p \geq 0)$ is the cross section integrated with $\sqrt{q^2} \leq 3 \text{ GeV}$ and $\cos \theta_p \geq 0$

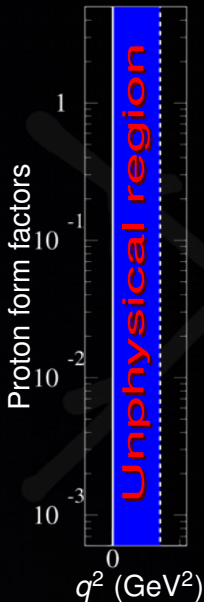
THE UNPHYSICAL REGION



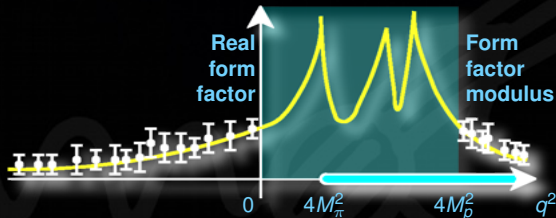
THE UNPHYSICAL REGION





THE UNPHYSICAL REGION



Unphysical region goes from $q^2 = 0$ up to the physical threshold $q^2 = 4M_p^2$



In that region, form factors

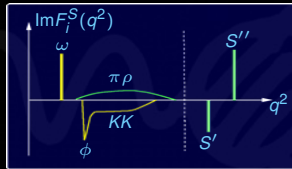
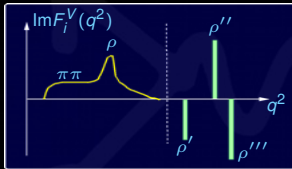
-  are still well defined but not (directly) experimentally accessible
-  are complex and, following VMD-based models, receive their main contribution from intermediate resonances




HANDLING THE UNPHYSICAL REGION₁

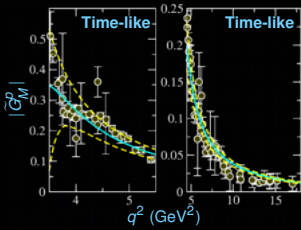
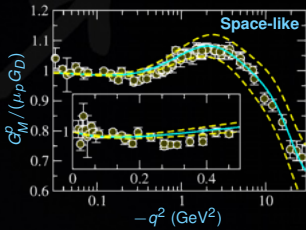
Model dependent disclosing [Höler, Mergell, Meissner, Hammer]


Optical theorem $\text{Im}\langle \bar{N}(\rho') N(\rho) | j^\mu | 0 \rangle \sim \sum_n \langle \bar{N}(\rho') N(\rho) | j^\mu | n \rangle \langle n | j^\mu | 0 \rangle$

Dispersion relations for the imaginary part $F(q_{\text{SL}}^2) = \frac{1}{\pi} \int_{4M_\pi^2}^{\infty} \frac{\text{Im}F(q_{\text{TL}}^2)}{q_{\text{TL}}^2 - q_{\text{SL}}^2} dq_{\text{TL}}^2$




-  2π and $2K$ continua are known
-  The ρ resonance with finite width
-  Dirac delta poles for higher mass states



-  Super convergence relations

$$\int_{4M_\pi^2}^{\infty} \text{Im} F_{1,2}(q^2) dq^2 = 0$$

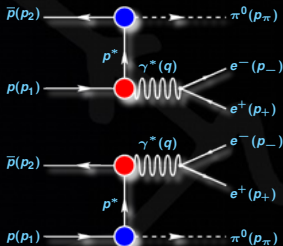
$$\int_{4M_\pi^2}^{\infty} q^2 \text{Im} F_2(q^2) dq^2 = 0$$
-  Asymptotic behaviors from perturbative QCD

HANDLING THE UNPHYSICAL REGION₂

Accessing the unphysical region

[C. Adamuscin, E.A. Kuraev, E. Tomasi-Gustafsson, F. Maas]

The initial state π -production
 $p\bar{p} \rightarrow \pi^0 e^+ e^-$



The process $p\bar{p} \rightarrow \pi^0 e^+ e^-$



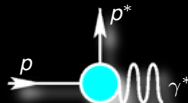
Described in general by **six** amplitudes which depend on **three** kinematical variables

Hadronic current [PRC75 045205]

$$J_\mu = \phi_\pi(p_\pi) \bar{v}(p_2) O_\mu u(p_1)$$

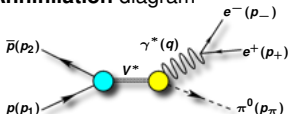
$$O_\mu = O_\mu[\Gamma_\mu(q)]$$

$$\langle N(p') | \Gamma_\mu(q) | N(p) \rangle = \bar{u}(p') \left[F_1(q^2) \gamma_\mu + \frac{i \sigma_{\mu\nu} q^\nu}{4M_p^2} F_2(q^2) \right] u(p)$$



Background:

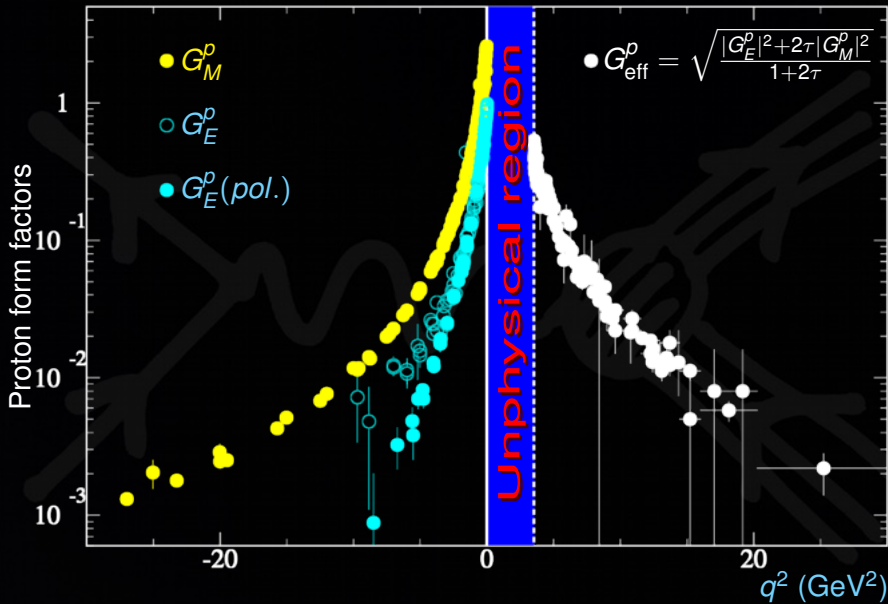
Annihilation diagram



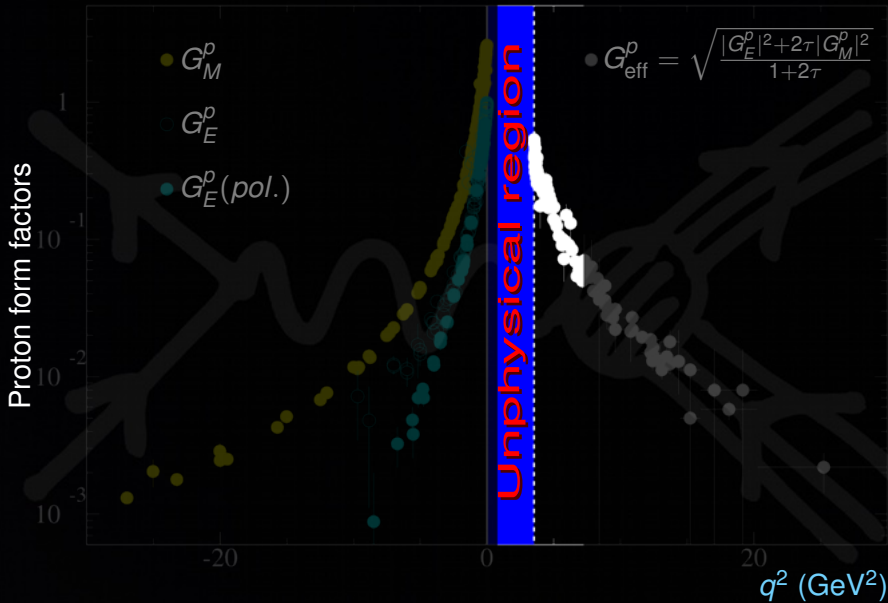
Polarization observables help in disentangle reaction mechanisms

[E. A. Kuraev *et al.*, J. Exp. Theor. Phys. 115 (2012) 93
 G.I. Gakh, E. Tomasi-Gustafsson, A. Dbeyssi, A.G. Gakh
 PhysRevC86 (2012) 025204]

THE THRESHOLD REGION



THE THRESHOLD REGION



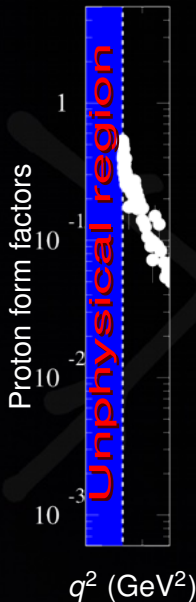
THE THRESHOLD REGION



Annihilation cross section

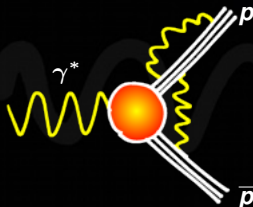
$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2 \beta C}{4q^2} \left[(1 + \cos^2 \theta) |G_M|^2 + \frac{1}{\tau} \sin^2 \theta |G_E|^2 \right]$$

THE THRESHOLD REGION



Annihilation cross section

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2 \beta C}{4q^2} \left[(1 + \cos^2 \theta) |G_M|^2 + \frac{1}{\tau} \sin^2 \theta |G_E|^2 \right]$$



Enhancement factor $\mathcal{E} = \frac{\pi\alpha}{\beta}$

It is responsible for the one-photon exchange $p\bar{p}$ final state interaction, dominates at threshold and cancels the phase-space factor.

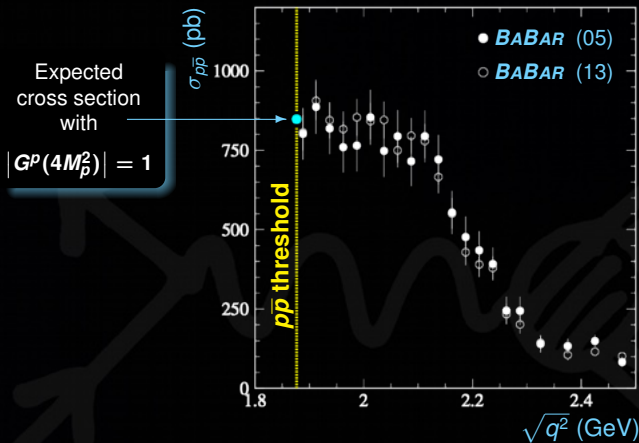
Resummation factor $\mathcal{R} = \frac{1}{1 - e^{-\frac{\pi\alpha}{\beta}}}$

It is responsible for the multi-photon $p\bar{p}$ final state interaction, becomes ineffective few MeV above threshold and accounts also for gluon exchange.

$$C = \mathcal{E} \times \mathcal{R} = \frac{\pi\alpha/\beta}{1 - \exp(-\pi\alpha/\beta)} \xrightarrow{\beta \rightarrow 0} \frac{\pi\alpha}{\beta} = \mathcal{E}$$

STEP AND PLATEAU IN *BABAR* DATA

Eur. Phys. J. A39 (2009) 315



At threshold

$$\sigma_{p\bar{p}}(4M_p^2) = \frac{\pi^2 \alpha^3}{2M_p^2} \frac{\beta_p}{\beta_p} |G^p(4M_p^2)|^2$$

$$\sigma_{p\bar{p}}(4M_p^2) = 0.85 |G^p(4M_p^2)|^2 \text{ nb}$$

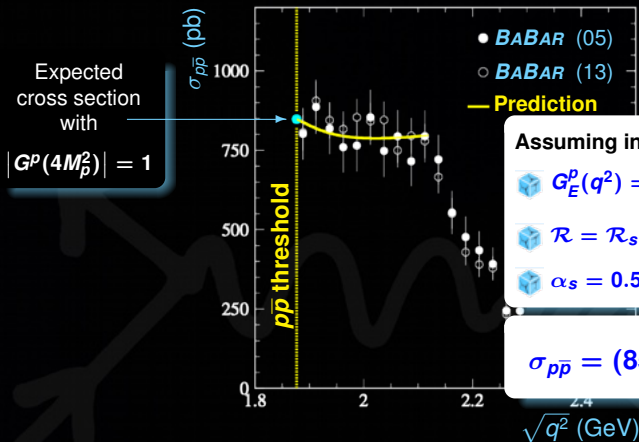


$$|G_S^p(4M_p^2)| \equiv 1$$

as pointlike fermion pairs!

STEP AND PLATEAU IN *BABAR* DATA

Eur. Phys. J. A39 (2009) 315



Assuming in $q^2 \leq (2.1\text{GeV})^2$:

$$G_E^p(q^2) = G_M^p(q^2) = 1$$

$$\mathcal{R} = \mathcal{R}_s = \left(1 - e^{-\frac{\pi\alpha_s}{\beta}}\right)^{-1}$$

$$\alpha_s = 0.5$$

$$\sigma_{p\bar{p}} = (850 \text{ pb}) \frac{1}{\tau} \mathcal{R}_s$$

At threshold

$$\sigma_{p\bar{p}}(4M_p^2) = \frac{\pi^2 \alpha^3}{2M_p^2} \frac{\beta_p}{\beta} |G^p(4M_p^2)|^2$$

$$\sigma_{p\bar{p}}(4M_p^2) = 0.85 |G^p(4M_p^2)|^2 \text{ nb}$$

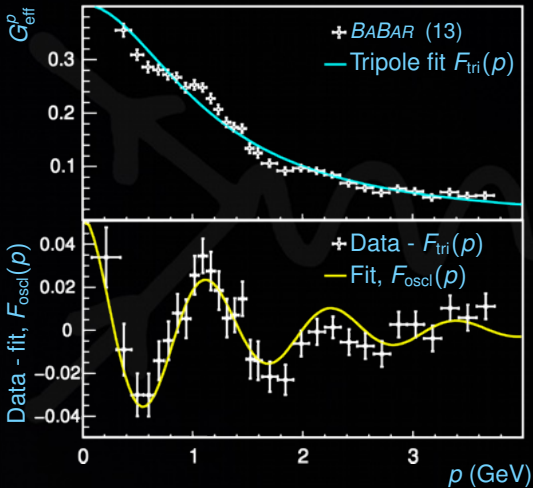


$$|G_S^p(4M_p^2)| \equiv 1$$

as pointlike fermion pairs!

PERIODIC INTERFERENCE NEAR THRESHOLD

A. BIANCONI, E. TOMASI-GUSTAFSSON, PHYS. REV. LETT. 114, 232301



$$F_{\text{oscl}}(p) = Ae^{-Bp} \cos(Cp + D)$$

$$A \ll 1$$

B damp. par.

$$C = r < 1 \text{ fm}$$

D thresh. shift

p is the momentum of the proton in the anti-proton rest frame.

The periodical behavior suggests an interference due to a rescattering of proton and antiproton at low kinetic energy and separation ~ 1 fm.

Proton and antiproton interact when they are almost phenomenological.

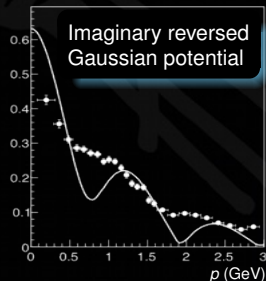
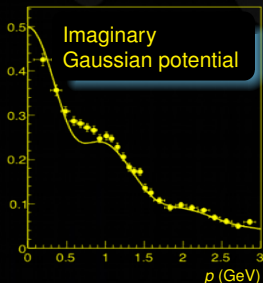
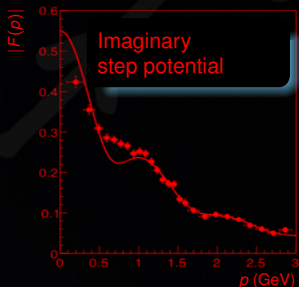
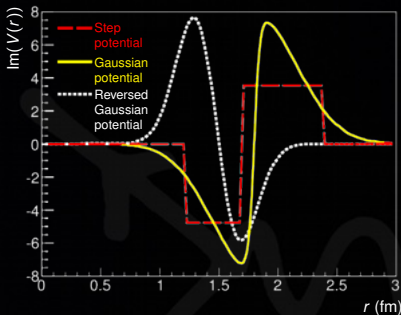
Unitarity implies a large imaginary part of form factors.

DOUBLE HOLLOW POTENTIALS

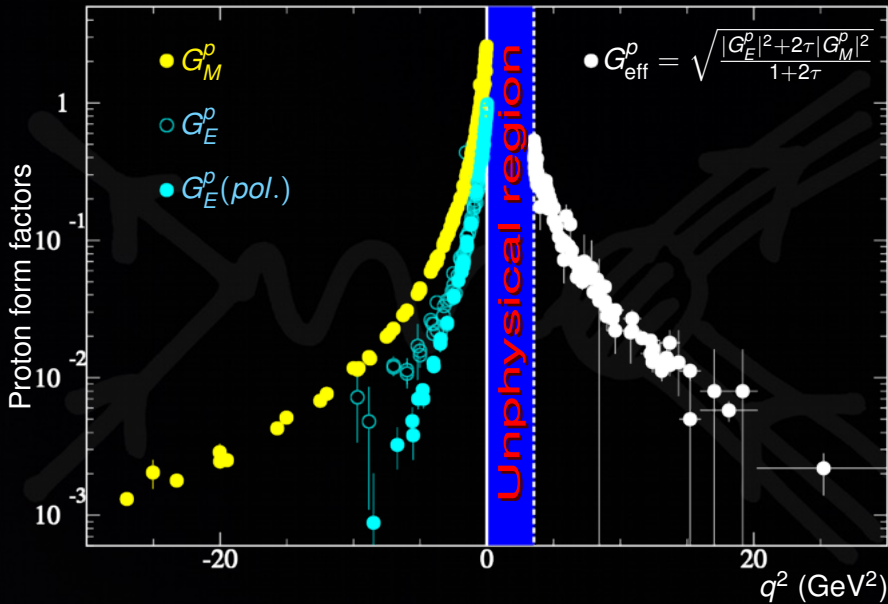
A. BIANCONI, E. TOMASI-GUSTAFSSON, PRC93, 035201

Double-layer optical potentials are phenomenologically required

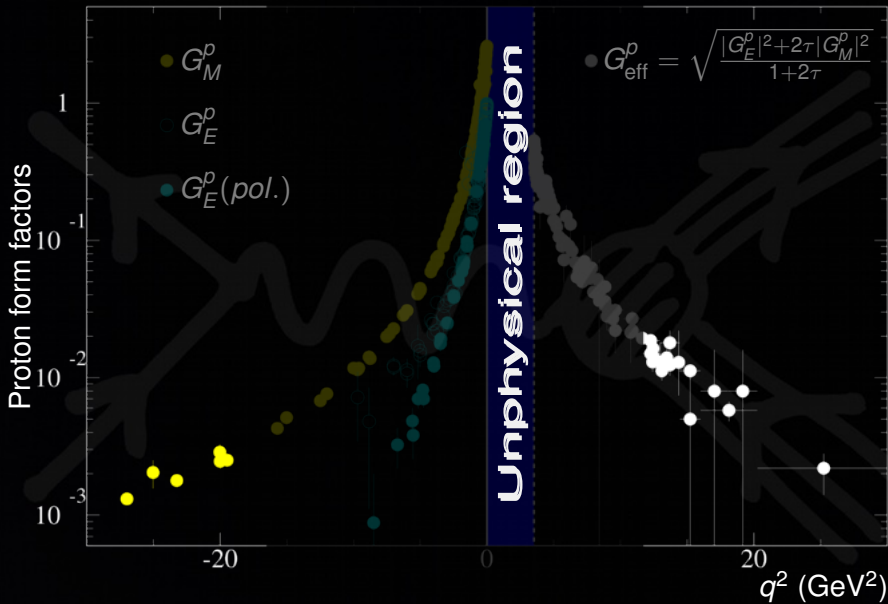
- ☒ Purely absorptive at $r > 1.7$ fm
- ☒ Flux-generating at $r < 1.7$ fm
- ☒ Quick (< 2 fm) transition absorption to flux-generating to get the period
- ☒ Maximum at $p = 0$
⇒ threshold enhancement
- ☒ Reversed potentials reproduce the oscillation amplitude **not the period**



THE ASYMPTOTIC REGIONS₁

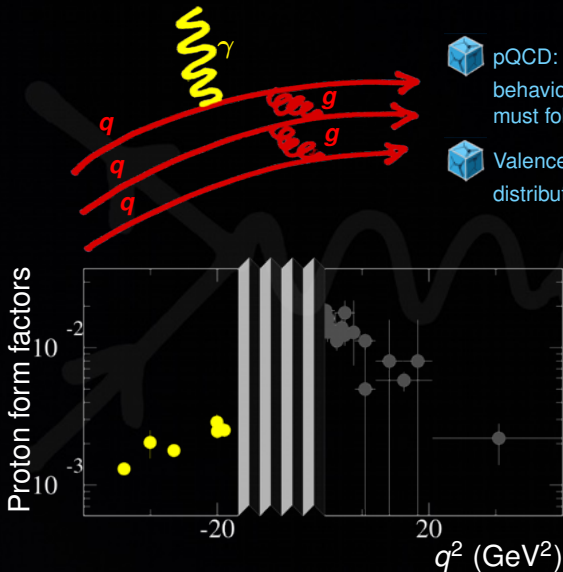


THE ASYMPTOTIC REGIONS₁



THE ASYMPTOTIC REGIONS₁

Space-like dimensional scaling



pQCD: as $q^2 \rightarrow -\infty$, asymptotic behaviors of F_1 and F_2 , and G_E and G_M must follow counting rules



Valence quarks exchange gluons to distribute the momentum transfer q



Dirac and Pauli form factors

$$F_i(q^2) \underset{q^2 \rightarrow -\infty}{\sim} (-q^2)^{-1-i}$$



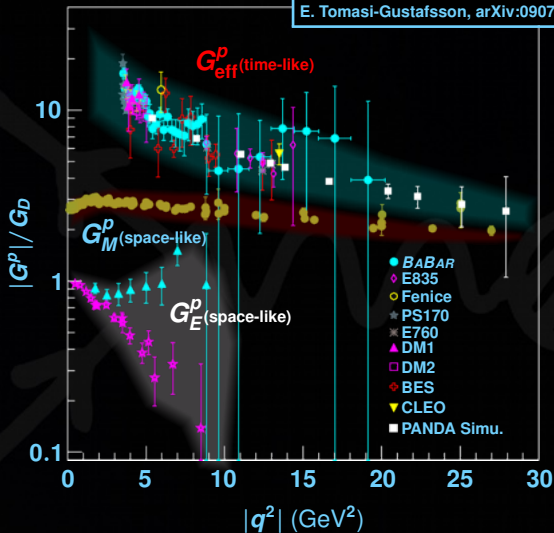
Sachs form factors

$$G_{E,M}(q^2) \underset{q^2 \rightarrow -\infty}{\sim} (-q^2)^{-2}$$

$$\frac{G_E(q^2)}{G_M(q^2)} \underset{q^2 \rightarrow -\infty}{\sim} \text{const.}$$

THE ASYMPTOTIC REGIONS₂

E. Tomasi-Gustafsson and M. P. Rekalo, PLB504,291
E. Tomasi-Gustafsson, arXiv:0907.4442



————— pQCD —————

$$G_{\text{eff}}^p(q^2) \underset{q^2 \rightarrow \infty}{\sim} G_M^p(q^2)$$

————— Phragmèn Lindelöf —————

$$\lim_{q^2 \rightarrow \infty} \frac{G_{\text{eff}}^p(q^2)}{G_M^p(-q^2)} = 1$$

—————

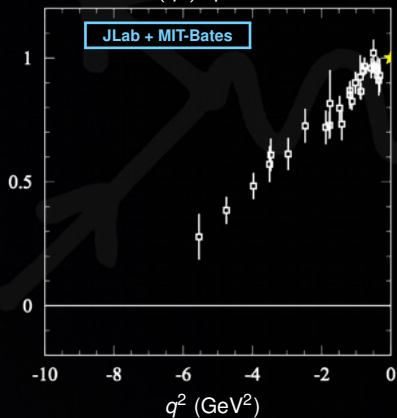
$$\text{ANALYTIC } R = \mu G_E / G_M$$

Eur. Phys. J. A32 (2007) 421

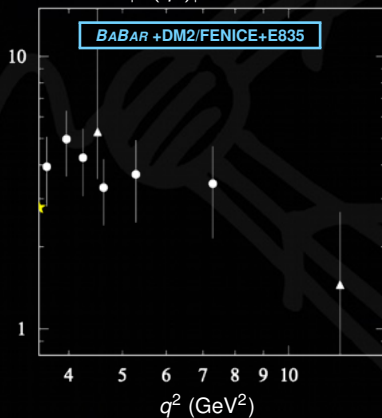
$$R(q^2) = R(0) + \frac{q^2}{\pi} \int_{4M_\pi^2}^{\infty} \frac{\text{Im}R(s)}{s(s - q^2)} ds$$



$R(q^2)$ space-like



$|R(q^2)|$ time-like



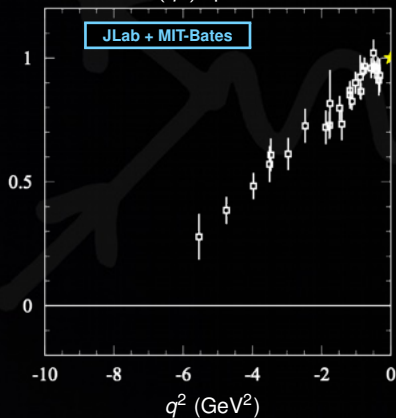
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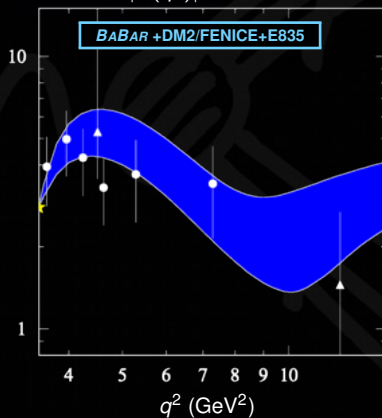
$$R(q^2) = R(0) + \frac{q^2}{\pi} \int_{4M_\pi^2}^{\infty} \frac{\text{Im}R(s)}{s(s - q^2)} ds$$

$\text{Re}q^2$

$R(q^2)$ space-like



$|R(q^2)|$ time-like



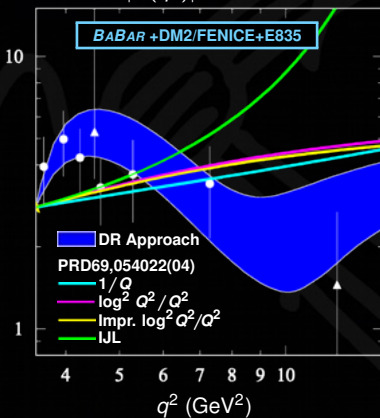
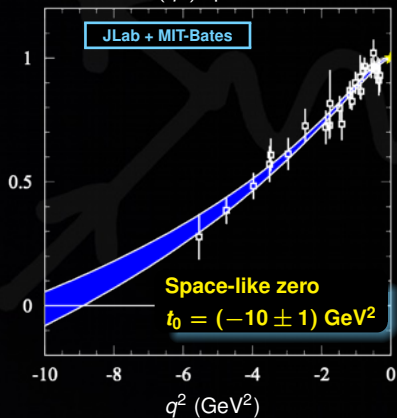
ANALYTIC $R = \mu G_E / G_M$

$$R(q^2) = R(0) + \frac{q^2}{\pi} \int_{4M_\pi^2}^{\infty} \frac{\text{Im}R(s)}{s(s - q^2)} ds$$

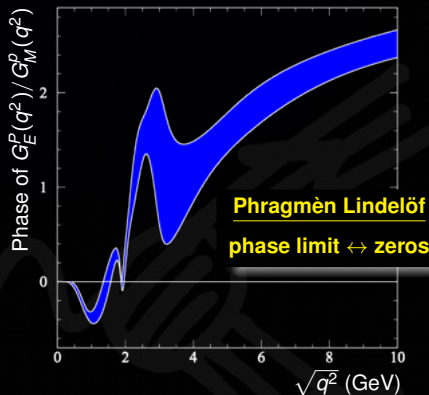
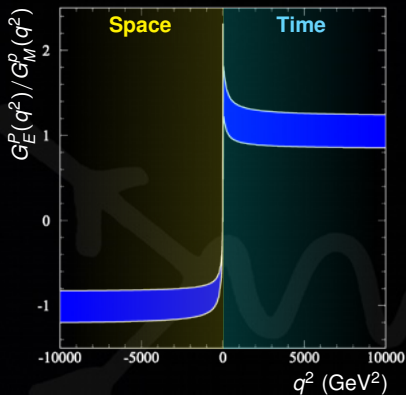


$R(q^2)$ space-like

$|R(q^2)|$ time-like



ASYMPTOTIC G_E/G_M AND PHASE



pQCD prediction

$$\frac{G_E^p(q^2)}{G_M^p(q^2)} \xrightarrow{|q^2| \rightarrow \infty} -1$$

Phase from DR

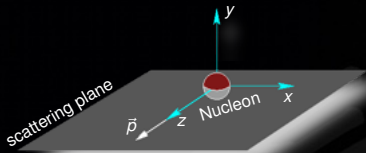
$$\phi(q^2) = -\frac{\sqrt{q^2 - s_{th}}}{\pi} \text{Pr} \int_{s_{th}}^{\infty} \frac{\ln |R(s)| ds}{\sqrt{s - s_{th}}(s - q^2)}$$

POLARIZATION FORMULAE IN THE TIME-LIKE REGION

The ratio $R(q^2)$ is complex for $q^2 \geq 4M_\pi^2$

$$R(q^2) = \mu_p \frac{G_E(q^2)}{G_M(q^2)} = |R(q^2)| e^{i\rho(q^2)}$$

The polarization depends on the phase ρ



[A.Z. Dubnickova, S. Dubnicka, M.P. Rekalo, NCA109,241(96)]

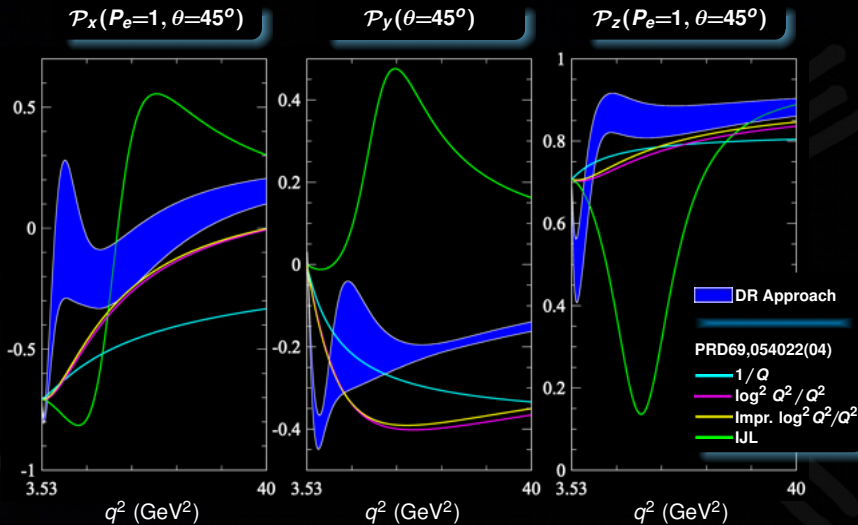
$$\mathcal{P}_y = - \frac{\sin(2\theta) |R| \sin(\rho)}{D\sqrt{\tau}} = \frac{d\sigma^\uparrow - d\sigma^\downarrow}{d\sigma^\uparrow + d\sigma^\downarrow} \equiv \mathcal{A}_y \left. \vphantom{\frac{\sin(2\theta) |R| \sin(\rho)}{D\sqrt{\tau}}} \right\} \text{Does not depend on } P_e$$

$$\mathcal{P}_x = - P_e \frac{2 \sin(2\theta) |R| \cos(\rho)}{D\sqrt{\tau}}$$

$$\mathcal{P}_z = P_e \frac{2 \cos(\theta)}{D} \left. \vphantom{\frac{2 \cos(\theta)}{D}} \right\} \text{Does not depend on } \rho$$

$$D = \frac{1 + \cos^2 \theta + \frac{1}{\tau} |R|^2 \sin^2 \theta}{\mu}, \quad \tau = \frac{q^2}{4M^2}, \quad P_e = \text{electron polarization}$$

SINGLE POLARIZATION



CONCLUSIONS PHYS. REV. 550-551 (2015) 1

Global models in space and time-like regions for proton and neutron, electric and magnetic form factors must be encouraged.

They can help in understanding:

- the threshold behavior
- the proton radius
- the presence of zeros
- the asymptotic behavior
- the unphysical region
- ...

 Jefferson Lab



To measure:

- zero of G_E^p in space-like region
- moduli of G_E and G_M in time-like region
- complex structure of form factors (polarization)
- unphysical time-like form factors ($p\bar{p} \rightarrow \pi^0 e^+ e^-$)
- ...

EXPERIMENTS: NOW AND FUTURE

See next talks by
Vladimir Druzhinin
Cristina Morales
Iris Zimmermann



- G_E^n at $-q^2 = 1.5 \text{ GeV}^2$ (Pol. ^3He)
- G_E^p and G_M^p for $-q^2 \leq 2.0 \text{ GeV}^2$

Mainz

Space-like
region



- [Hall A] G_E^p / G_M^p up to 14 GeV^2
- [Hall A] G_M^n up to 18 GeV^2
- [Hall A] G_E^n / G_M^n up to 10.2 GeV^2
- [Hall B] G_M^n (deuterium) up to 14 GeV^2
- [Hall C] G_E^n up to 7 GeV^2



at VEPP-2000
 e^+e^- collider



$|G_{\text{eff}}^p|, |G_{\text{eff}}^n|$ (scan)
 $q^2 \leq (4 \text{ GeV})^2$

Time-like
region

BESIII



at BEPCII

e^+e^- collider

$|G_E^p|, |G_M^p|, |G_{\text{eff}}^n|$ (scan and ISR)
 $q^2 \leq (3.5 \text{ GeV})^2$



at FAIR
 $p\bar{p}$ collider

$|G_E^p|, |G_M^p|, G_E^p / G_M^p$ phase (\bar{p} polarization?)
 $(2.4 \text{ GeV})^2 \leq q^2 \leq (3.7 \text{ GeV})^2$



at SuperKEKB

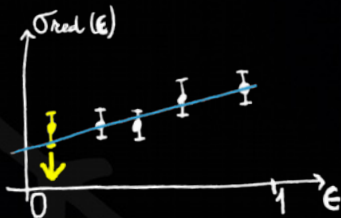
e^+e^- collider

?

$|G_E^p|, |G_M^p|$, (ISR)
 $q^2 \leq (4.5 \text{ GeV})^2$

Additional slides

THE ANDIVAHIS NORMALIZATION



Andivahis data on $\sigma_{\text{red}}(\epsilon)$ at lowest ϵ 's, $\{Q_f^2, \epsilon_{0j}, \sigma_{0j}, \delta\sigma_{0j}\}$, have been obtained with a different experimental setup.





The corresponding reduced cross section values are **normalized** in order to be aligned with the other points at the same Q^2 .

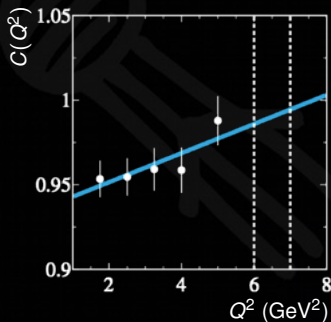
The normalization factor \bar{C} is obtained as the mean value of

$$C(Q_f^2) = \frac{G_{M0}^2 (R_0^2 \epsilon_{0,j} + |\tau_j|)}{\sigma_{0,j}}$$

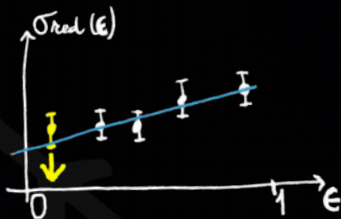
G_{M0}^2 and R_0^2 are the parameters obtained by not including $\{\epsilon_{0j}, \sigma_{0j}, \delta\sigma_{0j}\}$.

We find that:

-  the correction factor depends on Q^2
-  values $C(Q_f^2)$ are obtained by a linear fit
-  the correction, if needed, vanishes at high Q^2
-  the first two points are slightly affected by the procedure



THE ANDIVAHIS NORMALIZATION





Andivahis data on $\sigma_{\text{red}}(\epsilon)$ at lowest ϵ 's, $\{Q_j^2, \epsilon_{0j}, \sigma_{0j}, \delta\sigma_{0j}\}$, have been obtained with a different experimental setup.

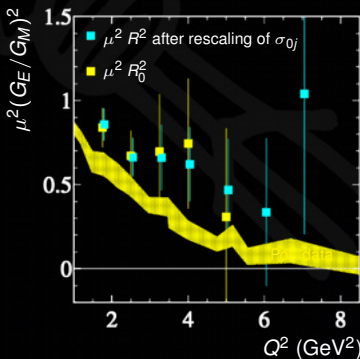
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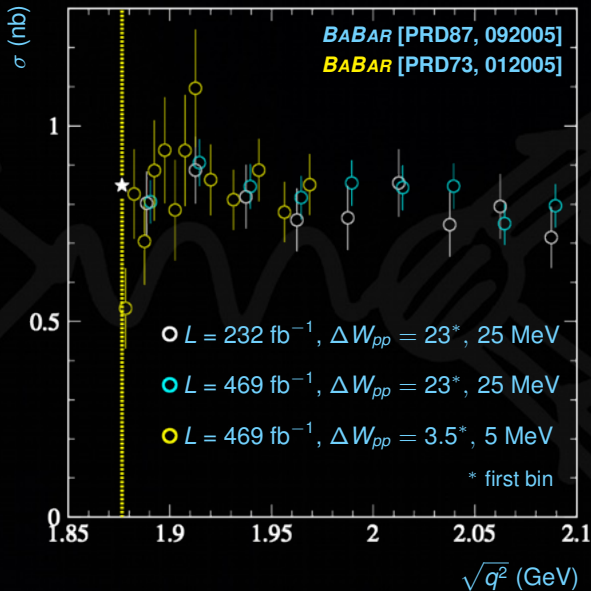
$$C(Q_j^2) = \frac{G_{M0}^2 (R_0^2 \epsilon_{0,j} + |\tau_j|)}{\sigma_{0,j}}$$

G_{M0}^2 and R_0^2 are the parameters obtained by not including $\{\epsilon_{0j}, \sigma_{0j}, \delta\sigma_{0j}\}$.

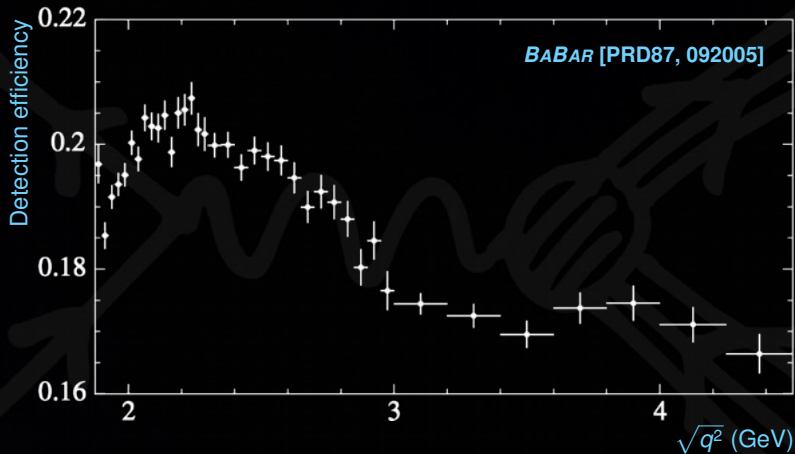
-  The correction does not change the tendency
-  Values of R^2 at high Q^2 do not follow the scaling hypothesis



THE THRESHOLD REGION₃



THE THRESHOLD REGION₃



PARTIAL WAVE FORM FACTORS



$$\begin{cases} P_\gamma = -1 \\ J_\gamma = 1 \end{cases} \quad \begin{cases} P_{p\bar{p}} = (-1)^{L+1} \\ S_{p\bar{p}} = 0, 1 \end{cases}$$



$$\begin{cases} L_{p\bar{p}} = 0, 2 \\ S_{p\bar{p}} = 1 \end{cases}$$

~~$$\begin{cases} L_{p\bar{p}} = 1 \\ S_{p\bar{p}} = 0 \end{cases}$$~~

Partial wave form factors

$$G_S^p = \frac{1}{3} \left(2G_M^p \sqrt{\frac{q^2}{4M_p^2}} + G_E^p \right)$$

$$G_D^p = \frac{1}{3} \left(G_M^p \sqrt{\frac{q^2}{4M_p^2}} - G_E^p \right)$$

Cross section

$$\sigma(q^2) = 2\pi\alpha^2\beta \frac{4M_p^2}{(q^2)^2} \left[C |G_S^p(q^2)|^2 + 2|G_D^p(q^2)|^2 \right]$$

ENHANCEMENT AND RESUMMATION FACTORS

Coulomb factor \mathcal{C} for S-wave only

Partial wave FF..... $G_S = (2G_M\sqrt{\tau} + G_E)/3$, $G_D = (G_M\sqrt{\tau} - G_E)/3$

Cross section..... $\sigma(q^2) = 2\pi\alpha^2 \frac{\beta}{\tau q^2} \left[\mathcal{C} |G_S(q^2)|^2 + 2|G_D(q^2)|^2 \right]$

Enhancement and Resummation factors

$$\mathcal{C} = \mathcal{E} \times \mathcal{R}$$

Enhancement factor

$$\mathcal{E} = \frac{\pi\alpha}{\beta}$$

- It is responsible for the **one-photon exchange $p\bar{p}$ FSI**

- dominates close to threshold: $\mathcal{C} \underset{\beta \sim 0}{\approx} \mathcal{E}$

- cancels the phase-space factor \Rightarrow

stepwise cross section at threshold

$$\sigma_{p\bar{p}}(4M_p^2) = \frac{\pi^2\alpha^3}{2M_p^2} \frac{\cancel{\beta}}{\cancel{\beta}} |G_S^p(4M_p^2)|^2 = 0.85 |G_S^p(4M_p^2)|^2 \text{ nb}$$

Resummation factor

$$\mathcal{R} = \frac{1}{1 - e^{-\frac{\pi\alpha}{\beta}}}$$

- It is responsible for the **multi-photon exchange $p\bar{p}$ FSI**

- becomes ineffective few MeV above threshold: $\mathcal{R} \underset{\beta > 0}{\approx} 1$

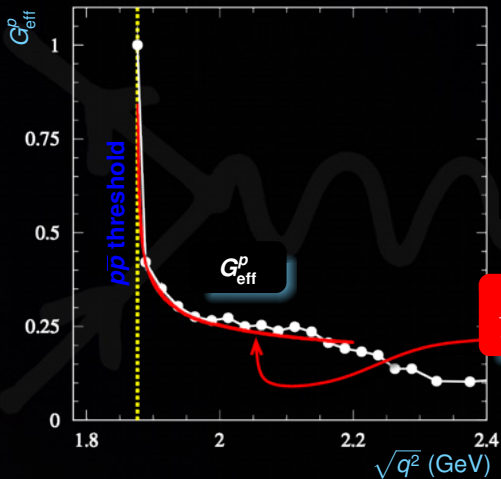
- must account also for gluon exchange

$$\mathcal{R} \rightarrow \mathcal{R}_s = \left[1 - \exp(-\pi\alpha_s/\tilde{\beta}) \right]^{-1}$$

$$\alpha_s \simeq 0.5$$

$$\tilde{\beta} = \beta/(1-\beta)$$

G_{eff}^p , WE GET WHAT WE PUT IN...



$$G_{\text{eff}}^p(q^2) = \sqrt{\frac{1}{\mathcal{R}} \frac{\sigma_{p\bar{p}}(q^2)}{\mathcal{E} \frac{16\pi\alpha^2}{3} \frac{\beta}{4q^2} \left(1 + \frac{1}{2\tau}\right)}}$$

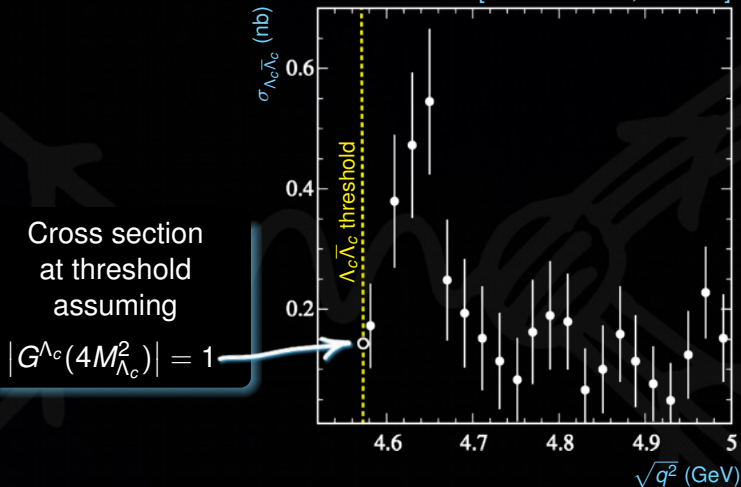
$$\mathcal{E} \frac{16\pi\alpha^2}{3} \frac{\beta}{4q^2} \left(1 + \frac{1}{2\tau}\right) \Big|_{\beta \rightarrow 0} \simeq \sigma_{p\bar{p}}(q^2)$$

$$G_{\text{eff}}^p(q^2) = \frac{1}{\sqrt{\mathcal{R}}}$$

$$\frac{1}{\sqrt{\mathcal{R}}} = \sqrt{1 - e^{-\frac{\pi\alpha}{\beta}}}$$

THE $e^+e^- \rightarrow \Lambda_c \bar{\Lambda}_c$ CROSS SECTION

[Belle PRL101, 172001]



BESIII is working on that
New results are coming soon

ISOTROPY AT THE $p\bar{p}$ PRODUCTION THRESHOLD

$$G_E(4M^2) = G_M(4M^2)$$

 Electromagnetic current:

$$J^\mu(p_1, p_2) = \bar{U}(p_2) \left[\gamma^\mu F_1(q^2) + \frac{i\sigma^{\mu\nu} q_\nu}{2M} F_2(q^2) \right] U(p_1)$$


$$\text{■ } F_1 = \frac{q^2 G_E - 4M^2 G_M}{q^2 - 4M^2} \quad \text{■ } F_2 = 4M^2 \frac{G_M - G_E}{q^2 - 4M^2}$$



F_1 and F_2 “can” be analytic (pointlike limit: $F_1(q^2) = 1$ and $F_2(q^2) = 0$)

 Annihilation cross section $\left[\tilde{G}_{E,M} \equiv G_{E,M}(4M^2) \right]$

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2 \beta C}{4q^2} \left[(1 + \cos^2\theta) |G_M|^2 + \frac{1}{\tau} \sin^2\theta |G_E|^2 \right] \xrightarrow[q^2 \rightarrow 4M^2]{|\tilde{G}_E| = |\tilde{G}_M|} \frac{\alpha^2 \beta C}{16M^2} \left[2|\tilde{G}_M|^2 \right]$$

 Partial wave form factors

$$\text{■ } G_S = \frac{2\sqrt{\frac{q^2}{4M^2}} G_M + G_E}{3} \xrightarrow[q^2 \rightarrow 4M^2]{} \tilde{G}_M \quad \text{■ } G_D = \frac{\sqrt{\frac{q^2}{4M^2}} G_M - G_E}{3} \xrightarrow[q^2 \rightarrow 4M^2]{} 0$$

ANISOTROPY AT THE PRODUCTION THRESHOLD

$$G_E(4M^2) \neq G_M(4M^2)$$

Dirac and Pauli form factors F_1 and F_2 are not analytic

To preserve G_E and G_M analyticity, F_1 and F_2 must have a **simple pole** at the threshold with **opposite residues**

$$F_1 = \frac{-4M^2 \Delta \tilde{G}}{q^2 - 4M^2} + F_1^{\text{an}}$$

$$F_2 = \frac{4M^2 \Delta \tilde{G}}{q^2 - 4M^2} + F_2^{\text{an}}$$

$$\Delta \tilde{G} \equiv \tilde{G}_E - \tilde{G}_M$$

$F_{1,2}^{\text{an}}$ is the analytic part of $F_{1,2}$

Annihilation cross section

$$\frac{d\sigma}{d\Omega} \xrightarrow{q^2 \rightarrow 4M^2} \frac{\alpha^2 \beta C}{8M^2} \left[|\tilde{G}_M|^2 + \text{Re} \left(\Delta \tilde{G} \tilde{G}_M^* \right) \sin^2 \theta \right]$$

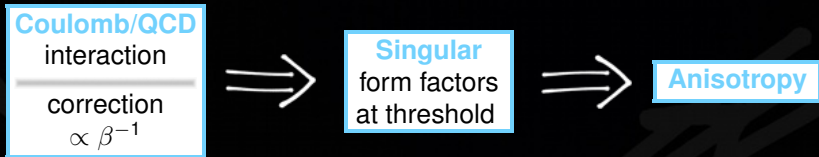
Assuming
 $|\Delta \tilde{G}| \ll |\tilde{G}_M|$

Partial wave form factors

$$G_S = \frac{2\sqrt{\frac{q^2}{4M^2}} G_M + G_E}{3} \xrightarrow{q^2 \rightarrow 4M^2} \tilde{G}_M + \frac{\Delta \tilde{G}}{3}$$

$$G_D = \frac{\sqrt{\frac{q^2}{4M^2}} G_M - G_E}{3} \xrightarrow{q^2 \rightarrow 4M^2} -\frac{\Delta \tilde{G}}{3}$$

SOURCES OF ANISOTROPY



Only Coulomb

Dmitriev, Milstein, PLB722 (13) 83

$$\tilde{G}_D \sim -\frac{\alpha^2}{8}$$

Very small
but not vanishing!

QCD Coulomb like

Brodsky, Hoang, Kuhn, Teubner,
PB359 (95) 355

**Large effect for
heavy quarks**

Anisotropy $\propto \beta^n$
No effect at threshold!

MEASURING ANISOTROPY AT THRESHOLD

$$e^+ e^- \rightarrow p \bar{p}$$

SND

CMD3

Very difficult

Efficiency drops with proton antiproton velocity

$$p \bar{p} \rightarrow e^+ e^-$$

PANDA

Very difficult

Normalization (Coulomb corrections...)

$$e^+ e^- \rightarrow p \bar{p} \gamma$$

BABAR

BESIII

Difficult

ISR technique: not enough statistics

$$e^+ e^- \rightarrow H_B \bar{H}_B$$

BESIII

Feasible with heavy baryons

The weak decay allows detection at threshold and polarization measurements (BESIII has...)