

Phenomenology of Nucleon Form Factors


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University of Science and Technology of China
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AGENDA

Nucleon Electromagnetic Form Factors

 Definition and properties


The space-like region

 Proton radius

 Rosenbluth versus polarization

The time-like region

 Unphysical region

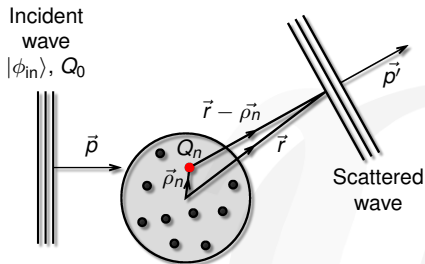
 Threshold

 An amazing effect

The asymptotic region

Conclusions

SEMI-CLASSICAL DEFINITION



Amplitude of the scattered wave

$$\mathcal{A}_n(\vec{r}) = Q_0 e^{i\vec{p}' \cdot \vec{r}} Q_n e^{i(\vec{p} - \vec{p}') \cdot \vec{\rho}_n}$$

$$\mathcal{A}(\vec{r}) = \sum_n \mathcal{A}_n(\vec{r}) = Q_0 e^{i\vec{p}' \cdot \vec{r}} \sum_n Q_n e^{i\vec{q} \cdot \vec{\rho}_n}$$

Form factor

$$F(\vec{q}) = \frac{1}{Q_0} \langle \phi_{in} | \sum_n Q_n e^{i\vec{q} \cdot \vec{\rho}_n} | \phi_{in} \rangle$$

The **unpolarized** differential cross section on an extended object

$$\frac{d\sigma}{d\Omega} = |F(\vec{q})|^2 \left(\frac{d\sigma}{d\Omega} \right)_{\text{pointlike}}$$

Form factor **modulus** is obtained by comparing theory and experiment

The Fourier transform of the form factor


$$\rho(\vec{r}) = \frac{Q_0}{(2\pi)^3} \int d^3\vec{q} F(\vec{q}) e^{i\vec{r} \cdot \vec{q}}$$

is the **charge spatial distribution**.


The form factor is Fourier transform of $\rho(\vec{r})$


$$F(\vec{q}) = \frac{1}{Q_0} \int d^3\vec{r} \rho(\vec{r}) e^{-i\vec{r} \cdot \vec{q}}$$

NUCLEON ELECTROMAGNETIC FORM FACTORS


 Form factors characterize the internal structure of a hadron
 $\Rightarrow \mathbf{F}_{\text{point-like}} = \text{constant.}$


 Elastic form factors contain information on the hadron ground state.

 In a parity and T -invariant theory, the electromagnetic structure of a particle of spin $S\hbar$ is defined by **$2S + 1$ form factors.**

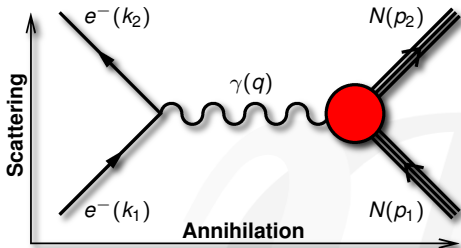
 Neutron and proton form factors are different.

 Playground for **theory** and **experiment**:

 at low q^2 probe the size of the hadron;

 at high q^2 test the QCD counting rule.

DIRAC AND PAULI FORM FACTORS



Scattering: $e^- N \rightarrow e^- N$
Space-like kinematic region

$$q^2 = -2\omega_1\omega_2(1 - \cos \theta_e) \leq 0$$



Annihilation: $e^+ e^- \leftrightarrow N\bar{N}$
Time-like kinematic region

$$q^2 = 4\omega^2 > 0$$

Scattering amplitude
in **Born** approximation

$$\mathcal{M} = \frac{1}{q^2} [e \bar{u}(k_2) \gamma_\mu u(k_1)] \underbrace{[e \bar{U}(p_2) \Gamma^\mu(p_1, p_2) U(p_1)]}_{\text{Nucleon EM 4-current: } \mathbf{J}_N^\mu}$$

From Lorenz and gauge invariance

$$\Gamma^\mu(p_1, p_2) = \gamma^\mu F_1^N(q^2) + \frac{i\sigma^{\mu\nu} q_\nu}{2M_N} F_2^N(q^2)$$



Dirac FF: $F_1^N(q^2)$, $F_1^N(0) = Q_N$



Pauli FF: $F_2^N(q^2)$, $F_2^N(0) = \kappa_N$

$Q_N = N$ electric charge

$\kappa_N = N$ anomalous magnetic moment

SACHS FORM FACTORS

Breit frame

No energy exchanged

$$p_1 = (E, -\vec{q}/2)$$

$$p_2 = (E, \vec{q}/2)$$

$$q = (0, \vec{q})$$

Nucleon electromagnetic four-current

$$J_N^\mu = (J_N^0, \vec{J}_N) \quad \left\{ \begin{array}{l} \rho_q = J_N^0 = e \left[F_1^N + \frac{q^2}{4M_N^2} F_2^N \right] \\ \vec{J}_N = e \bar{U}(p_2) \vec{\gamma} U(p_1) \left[F_1^N + F_2^N \right] \end{array} \right.$$

Sachs Nucleon Form Factors

$$G_M^N(q^2) = F_1^N(q^2) + F_2^N(q^2) \quad G_E^N(q^2) = F_1^N(q^2) + \frac{q^2}{4M_N^2} F_2^N(q^2)$$

In the Breit frame represent the **Fourier transforms** of charge and **magnetic moment spatial distributions** of the nucleon

Normalization at $q^2 = 0$

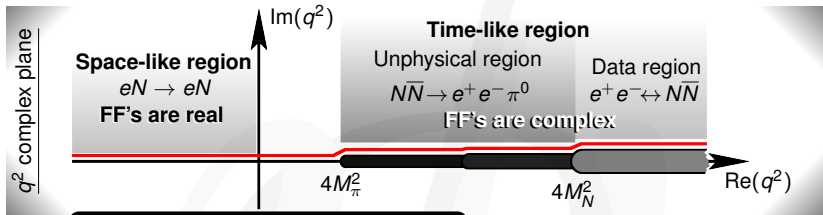
$$\int d^3x \psi^\dagger \psi = 1 \quad G_E^N(0) = \mathcal{Q}_N$$

$$\int d^3x \psi^\dagger \vec{x} \psi = 0 \quad G_M^N(0) = \mu_N$$

$$\mu_N = \mathcal{Q}_N + \kappa_N$$

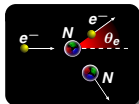
is the nucleon magnetic moment

CROSS SECTIONS AND ANALYTICITY



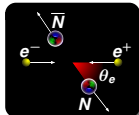
$$\text{Crossing: tot. helicity} = \begin{cases} 1 \Rightarrow G_E \\ 0 \Rightarrow G_M \end{cases}$$

$$G_E(4M_N^2) = G_M(4M_N^2)$$



Elastic scattering

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2 \omega_2 \cos^2 \frac{\theta_e}{2}}{4\omega_1^3 \sin^4 \frac{\theta_e}{2}} \left[G_E^2 - \tau \left(1 + 2(1-\tau) \tan^2 \frac{\theta_e}{2} \right) G_M^2 \right] \frac{1}{1-\tau} \quad \tau = \frac{q^2}{4M_N^2}$$

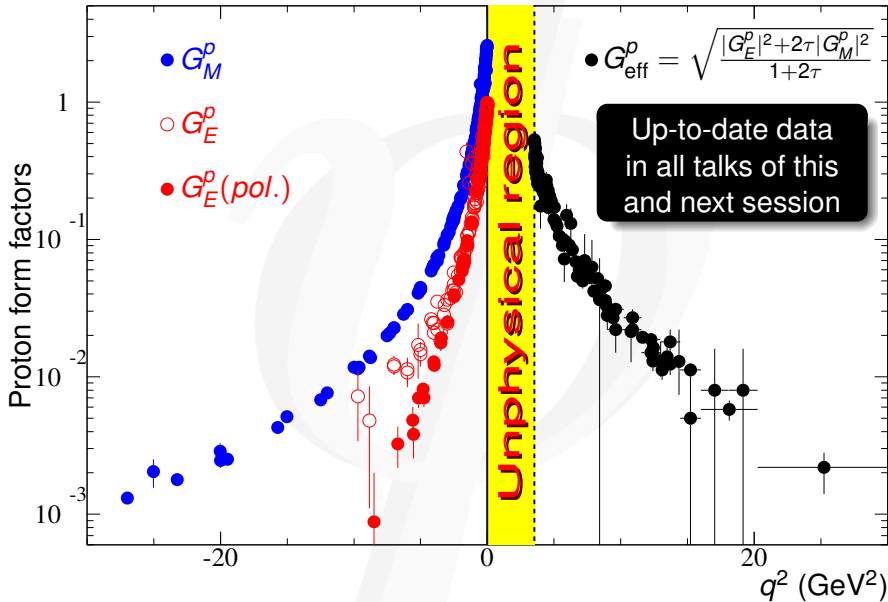


Annihilation

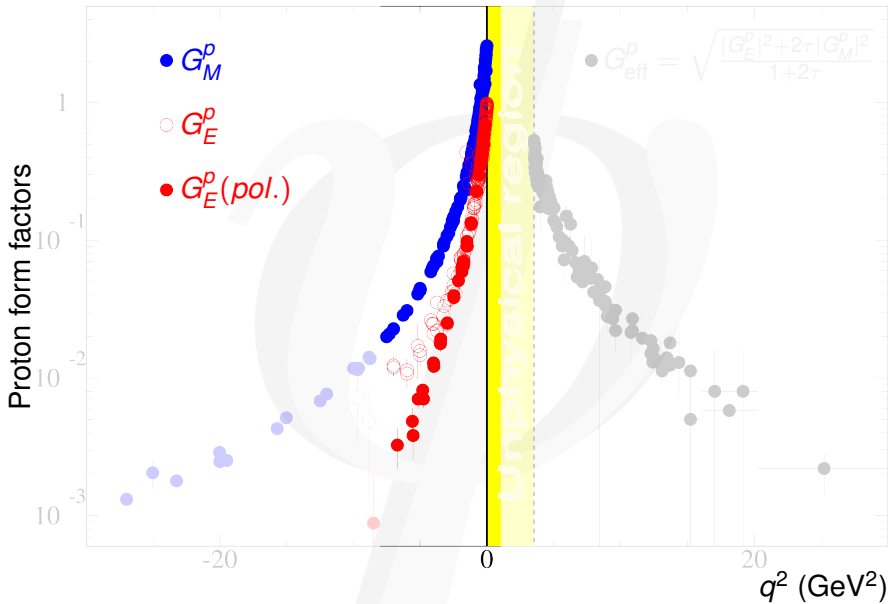
$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2 \beta C}{4q^2} \left[(1 + \cos^2 \theta) |G_M|^2 + \frac{1}{\tau} \sin^2 \theta |G_E|^2 \right]$$

$$\beta = \sqrt{1 - \frac{1}{\tau}}$$

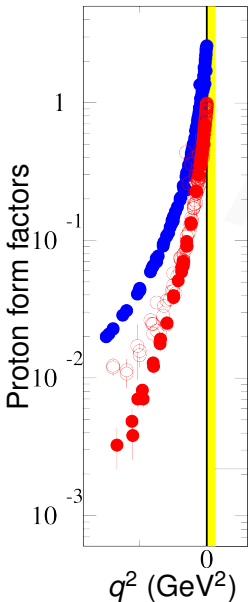
THE PROTON RADIUS



THE PROTON RADIUS



THE PROTON RADIUS



$$G_E^p(q^2) = \int d^3\vec{r} \rho(r) e^{i\vec{q}\cdot\vec{r}} = 1 + \frac{1}{6} q^2 \langle r_c^2 \rangle + \mathcal{O}(q^4)$$

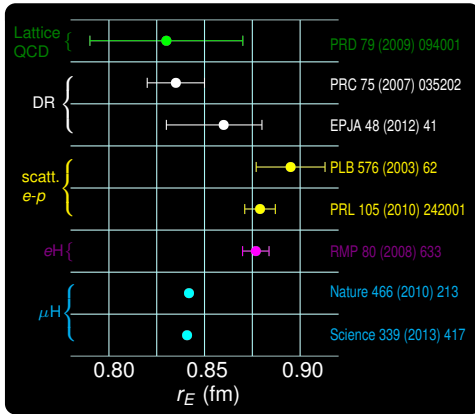
$\rho(r)$: normalized spherical charge density

The charge radius

$$r_E = \sqrt{\langle r_c^2 \rangle} = \sqrt{4\pi \int_0^\infty r^4 \rho(r) dr} = \sqrt{\frac{6}{G_E^p(0)} \left. \frac{dG_E^p}{dq^2} \right|_{q^2=0}}$$





Charge density $\rho(r)$	Form factor $G_E^p(q^2)$	Charge radius r_E	Comments
$\delta^3(r)$	1	0	pointlike
$e^{-\lambda r}$	$\lambda^4 / (q^2 + \lambda^2)^2$	$2\sqrt{3}/\lambda$	dipole
$e^{-\lambda r}/r$	$\lambda^2 / (q^2 + \lambda^2)$	$\sqrt{6}/\lambda$	monopole
$e^{-\lambda r^2}/r^2$	$e^{-r^2/(4\lambda^2)}$	$1/\sqrt{2\lambda}$	gaussian

THE PROTON RADIUS



Analyticity via **dispersion relations** and **QCD counting rules** can give directly the proton radius. . .

Ongoing discussions. . .

-  $q^2 \rightarrow 0^-$ extrapolation
-  Radiative corrections
-  Two-photon exchange
-  Coulomb corrections

Logarithmic derivative of form factor at $q^2 = 0$ by means of dispersion relations for the logarithm

$$r_E^2 = \frac{12M_\pi^2}{\pi} \int_{4M_\pi^2}^{\infty} \frac{\ln |G_E^p(t)/G_E^p(0)|}{t^2 \sqrt{t - 4M_\pi^2}} dt$$



Rosenbluth formula

$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega} \right)_{\text{Mott}} \frac{1}{1-\tau} \left[G_E^2 - \frac{\tau}{\epsilon} G_M^2 \right] \quad \tau = \frac{q^2}{4M_N^2}$$



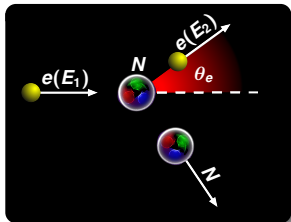
Mott pointlike cross section

$$\left(\frac{d\sigma}{d\Omega} \right)_{\text{Mott}} = \frac{4\alpha^2}{(-q^2)^2} \frac{E_2^3}{E_1} \cos^2(\theta_e/2)$$



Photon polarization

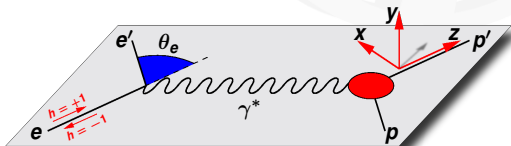
$$\epsilon = \left[1 + 2(1-\tau) \tan^2(\theta_e/2) \right]^{-1}$$



In case of **polarized electrons** ($h = \pm 1$) on unpolarized nucleon target:

$$P'_x = -\frac{2\sqrt{\tau(\tau-1)}}{G_E^2 - \frac{\tau}{\epsilon} G_M^2} G_E G_M \tan\left(\frac{\theta_e}{2}\right)$$

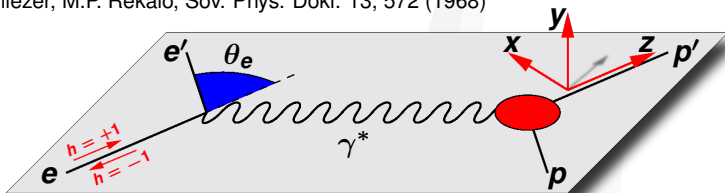
$$P'_z = \frac{(E_e + E'_e) \sqrt{\tau(\tau-1)}}{M(G_E^2 - \frac{\tau}{\epsilon} G_M^2)} G_M \tan^2\left(\frac{\theta_e}{2}\right)$$



$$\frac{P'_x}{P'_z} = -\frac{2M \cot(\theta_e/2)}{E_e + E'_e} \frac{G_E}{G_M}$$

POLARIZATION OBSERVABLES

A.I. Akhiezer, M.P. Rekalo, Sov. Phys. Dokl. 13, 572 (1968)



Elastic scattering of longitudinally polarized ($h = \pm 1$) electrons on nucleon target



Hadronic tensor: $W_{\mu\nu} = \underbrace{W_{\mu\nu}(0)}_{\text{no pol.}} + \underbrace{W_{\mu\nu}(\vec{P}) + W_{\mu\nu}(\vec{P}')}_{\text{ini. or fin. pol. of } N} + \underbrace{W_{\mu\nu}(\vec{P}, \vec{P}')}_{\text{ini. and fin. pol. of } N}$



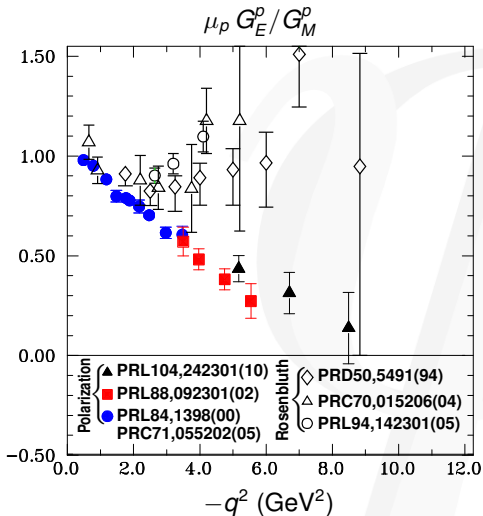
In case of polarized ($h = \pm 1$) electrons on unpolarized nucleon target:

$$P'_x = -\frac{2\sqrt{\tau(\tau-1)}}{G_E^2 - \frac{\tau}{\epsilon} G_M^2} G_E G_M \tan\left(\frac{\theta_e}{2}\right)$$

$$P'_z = \frac{(E_e + E'_e)\sqrt{\tau(\tau-1)}}{M(G_E^2 - \frac{\tau}{\epsilon} G_M^2)} G_M^2 \tan^2\left(\frac{\theta_e}{2}\right)$$

$$\frac{P'_x}{P'_z} = -\frac{2M \cot(\theta_e/2)}{E_e + E'_e} \frac{G_E}{G_M}$$

$\mu_p G_E^p / G_M^p$: ROSENBLUTH AND POLARIZATION TECHNIQUES



“Standard” dipole for the proton magnetic form factors G_M^p



Linear deviation from the dipole for the electric proton form factor G_E^p



QCD scaling still not reached



Zero crossing for G_E^p

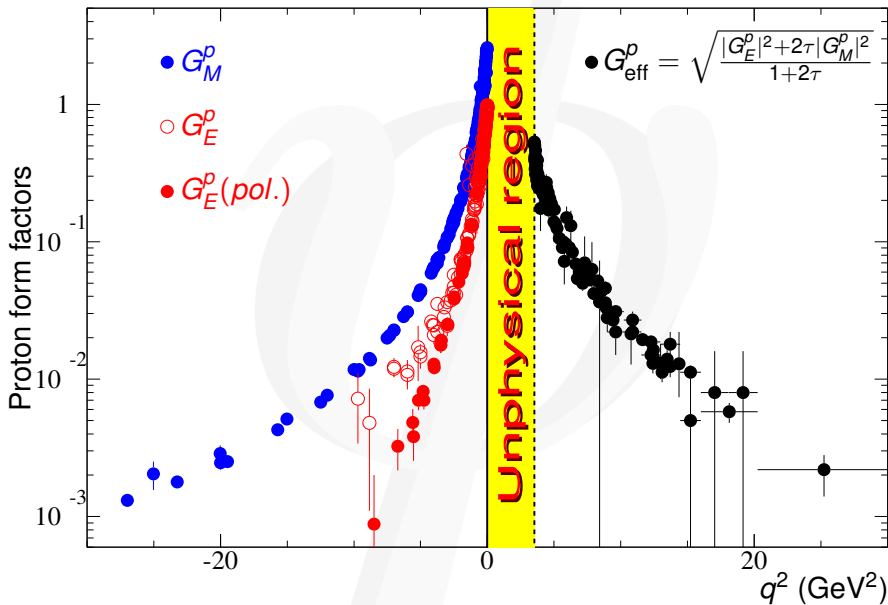


Polarization data do not agree with old Rosenbluth data (◇)

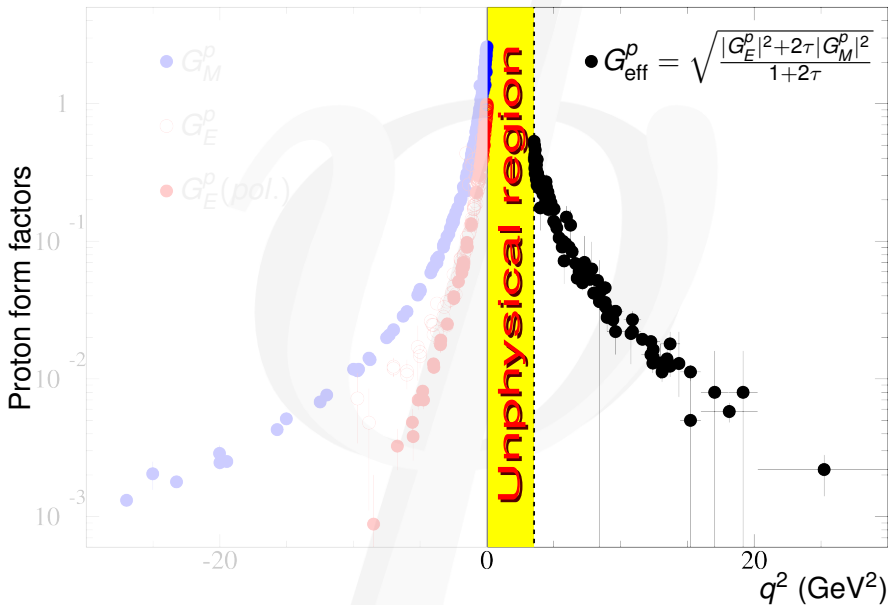


New Rosenbluth separation data from JLab **still do not agree** with polarization data

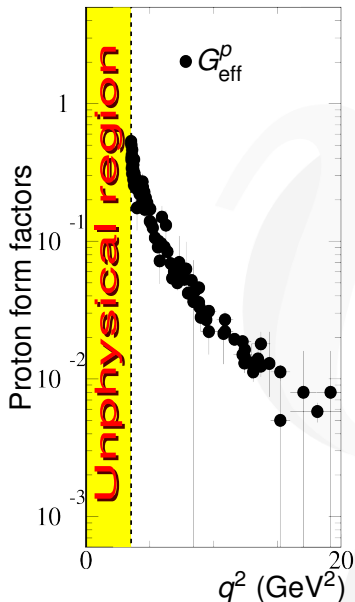
THE TIME-LIKE REGION



THE TIME-LIKE REGION



THE TIME-LIKE REGION



Differential cross section $e^+ e^- \rightarrow p\bar{p}$

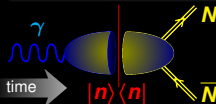
A. Zichichi, S. M. Berman, N. Cabibbo, R. Gatto [NC XXIV (1962) 170]

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2 \beta C}{4q^2} \left[(1 + \cos^2 \theta) |G_M^p|^2 + \frac{1}{\tau} \sin^2 \theta |G_E^p|^2 \right]$$

Optical theorem

$$\text{Im}(\overline{N}(p') N(p) |j^\mu| 0) \sim \sum_n \overline{N}(p') N(p) |j^\mu| n \langle n | j^\mu | 0 \rangle$$

$|n\rangle$ are on-shell intermediate states: $2\pi, 3\pi, 4\pi, \dots$



Form factors are complex for $q^2 > 4M_\pi^2$



The cross section is an **even function of $\cos \theta$**



It does **not depend on the form factor phases**



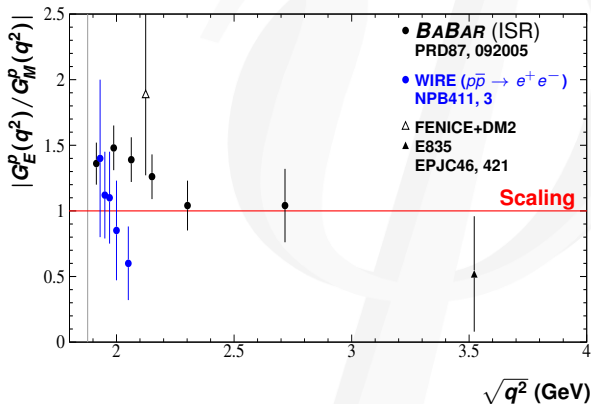
At high q^2 the $|G_E^p|^2$ contribution is suppressed



The **unphysical region is not accessible** through the annihilations $e^+ e^- \leftrightarrow p\bar{p}$

TIME-LIKE $|G_E^p/G_M^p|$ MEASUREMENTS

$$\frac{d\sigma}{d\cos\theta} = \frac{\pi\alpha^2\beta C}{2q^2} |G_M^p|^2 \left[(1 + \cos^2\theta) + \frac{4M_p^2}{q^2} \sin^2\theta \left| \frac{G_E^p}{G_M^p} \right|^2 \right]$$



$\gamma\gamma$ exchange

$C=+1$

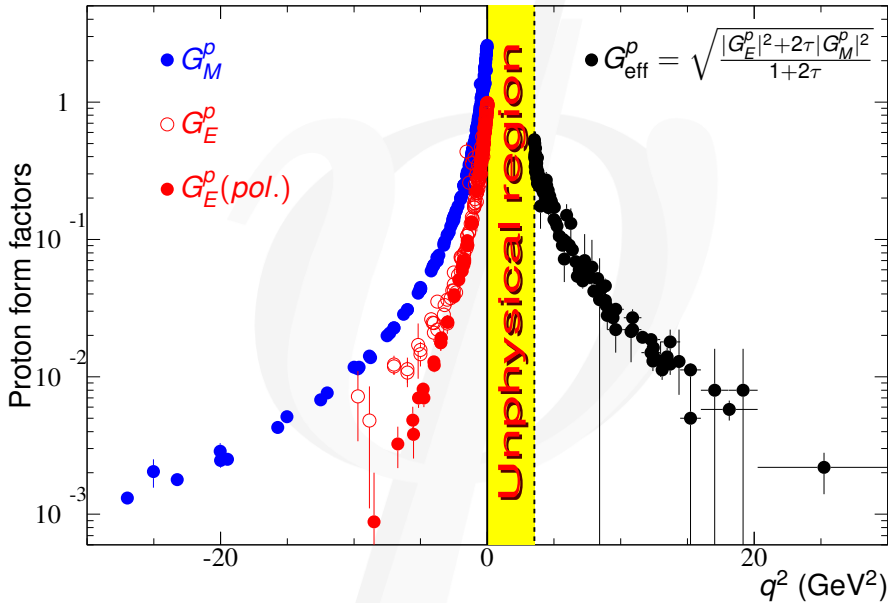
$\gamma\gamma$ exchange interferes with the Born term

↓

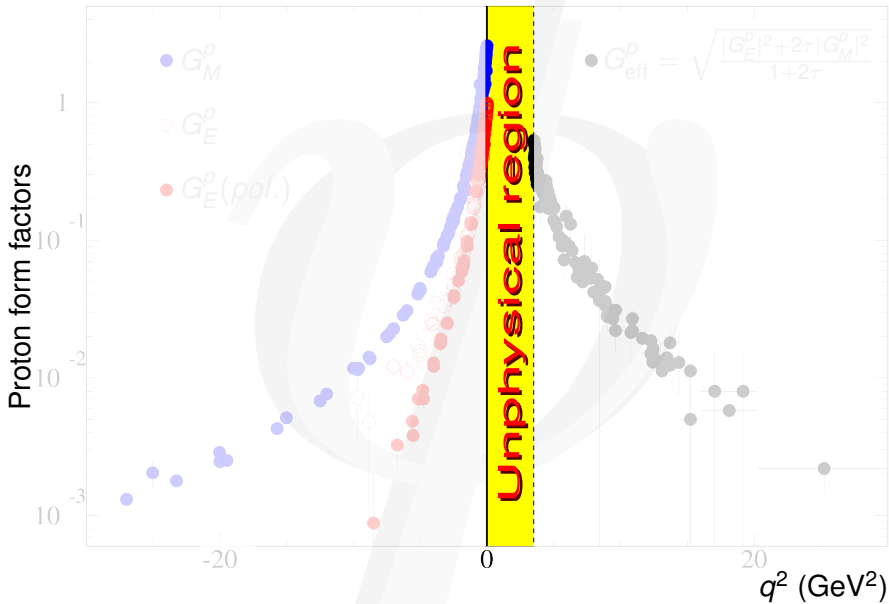
Asymmetry in angular distributions

[E. Tomasi-Gustafsson, G.I. Gakh, NPA771,169(06)]

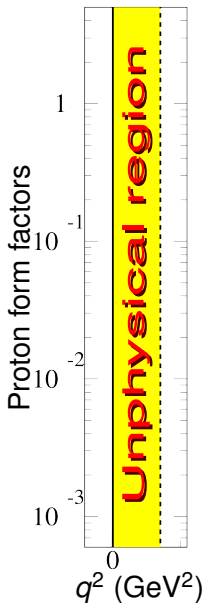
THE UNPHYSICAL REGION



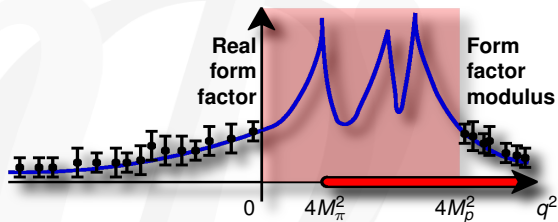
THE UNPHYSICAL REGION



THE UNPHYSICAL REGION



Unphysical region goes from $q^2 = 0$ up to the physical threshold $q^2 = 4M_p^2$



In that region, form factors



are still well defined but not (directly) experimentally accessible



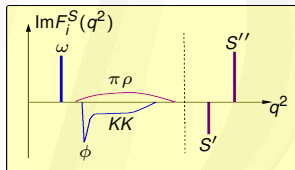
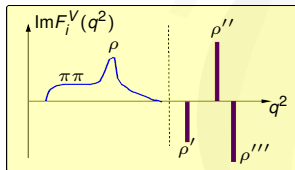
are complex and, following VMD-based models, receive their main contribution from intermediate resonances




HANDLING THE UNPHYSICAL REGION₁

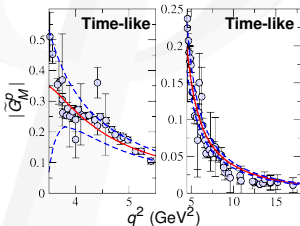
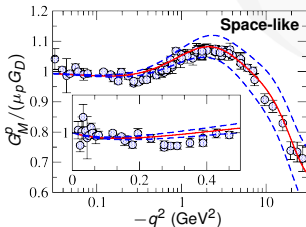
Model dependent disclosing [Höler, Mergell, Meissner, Hammer]


Optical theorem $\text{Im}\langle \bar{N}(p') N(p) | j^\mu | 0 \rangle \sim \sum_n \langle \bar{N}(p') N(p) | j^\mu | n \rangle \langle n | j^\mu | 0 \rangle$

Dispersion relations for the imaginary part $F(q_{\text{SL}}^2) = \frac{1}{\pi} \int_{4M_\pi^2}^{\infty} \frac{\text{Im}F(q_{\text{TL}}^2)}{q_{\text{TL}}^2 - q_{\text{SL}}^2} dq_{\text{TL}}^2$




-  2π and $2K$ continua are known
-  The ρ resonance with finite width
-  Dirac delta poles for higher mass states



-  Super convergence relations

$$\int_{4M_\pi^2}^{\infty} \text{Im} F_{1,2}(q^2) dq^2 = 0$$

$$\int_{4M_\pi^2}^{\infty} q^2 \text{Im} F_2(q^2) dq^2 = 0$$
-  Asymptotic behaviors from perturbative QCD

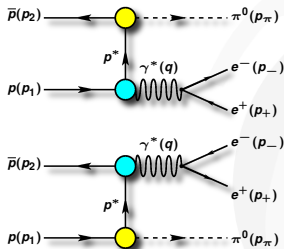
HANDLING THE UNPHYSICAL REGION₂

Accessing the unphysical region

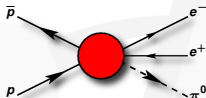
[C. Adamuscin, E.A. Kuraev, E. Tomasi-Gustafsson, F. Maas]

The initial state π -production

$$p\bar{p} \rightarrow \pi^0 e^+ e^-$$



The process $p\bar{p} \rightarrow \pi^0 e^+ e^-$



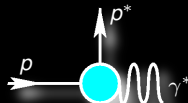
Described in general by **six** amplitudes which depend on **three** kinematical variables

Hadronic current [PRC75 045205]

$$J_\mu = \phi_\pi(p_\pi) \bar{v}(p_2) O_\mu u(p_1)$$

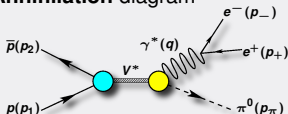
$$O_\mu = O_\mu[\Gamma_\mu(q)]$$

$$\langle N(p') | \Gamma_\mu(q) | N(p) \rangle = \bar{u}(p') \left[F_1(q^2) \gamma_\mu + \frac{i \sigma_{\mu\nu} q^\nu}{4M_p^2} F_2(q^2) \right] u(p)$$



Background:

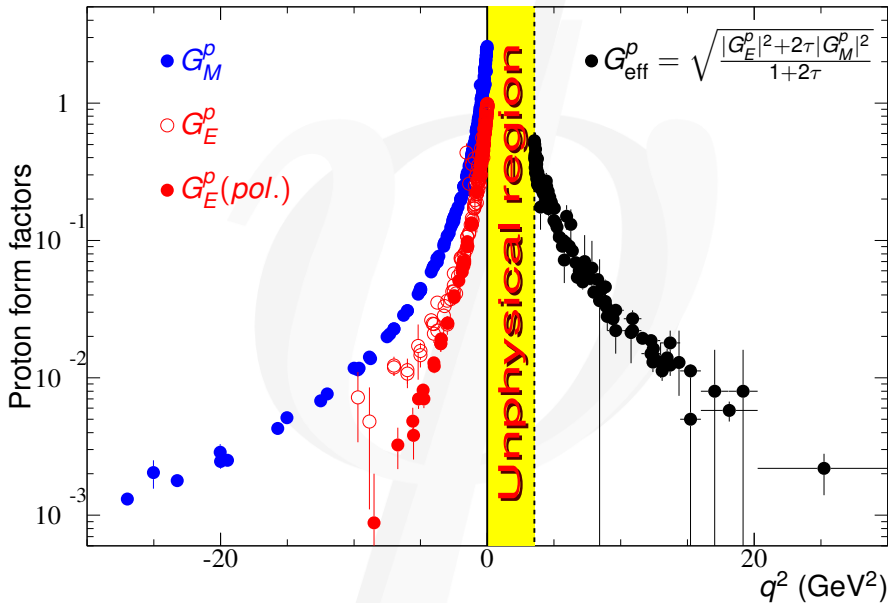
Annihilation diagram



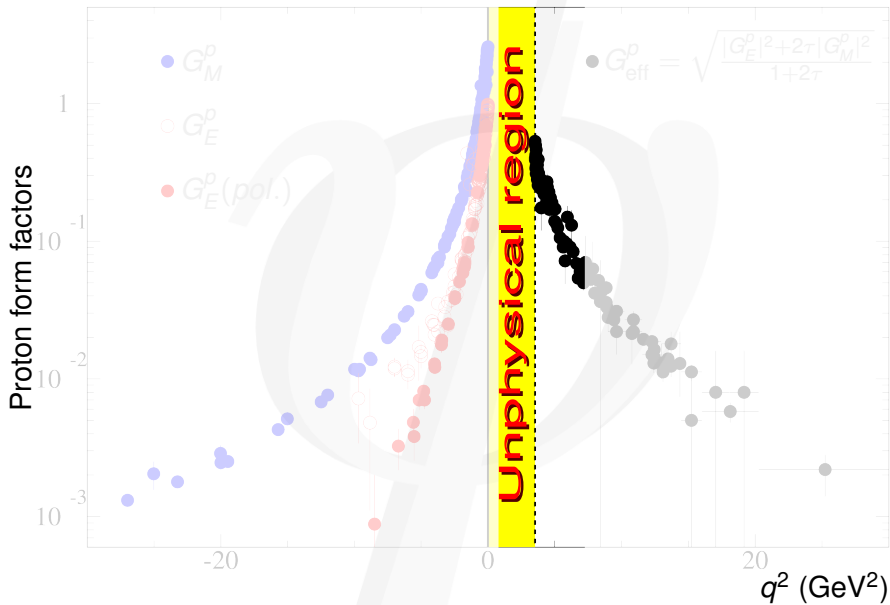
Polarization observables help in disentangle reaction mechanisms

[E. A. Kuraev *et al.*, J. Exp. Theor. Phys. 115 (2012) 93
G.I. Gakh, E. Tomasi-Gustafsson, A. Dbeyssi, A.G. Gakh
PhysRevC86 (2012) 025204]

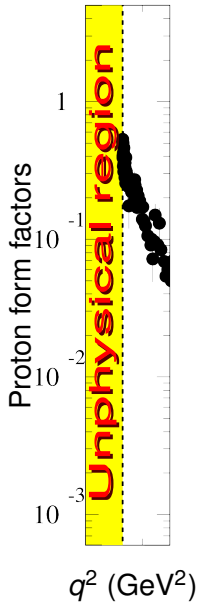
THE THRESHOLD REGION



THE THRESHOLD REGION



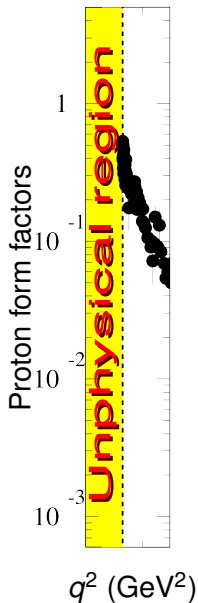
THE THRESHOLD REGION



Annihilation cross section

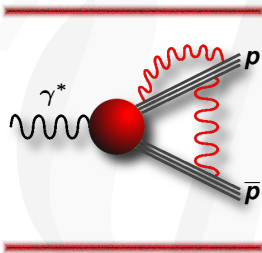
$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2 \beta C}{4q^2} \left[(1 + \cos^2 \theta) |G_M|^2 + \frac{1}{\tau} \sin^2 \theta |G_E|^2 \right]$$

THE THRESHOLD REGION



Annihilation cross section

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2 \beta \mathcal{C}}{4q^2} \left[(1 + \cos^2 \theta) |G_M|^2 + \frac{1}{\tau} \sin^2 \theta |G_E|^2 \right]$$



Enhancement factor $\mathcal{E} = \frac{\pi\alpha}{\beta}$

It is responsible for the one-photon exchange $p\bar{p}$ final state interaction, dominates at threshold and cancels the phase-space factor.

Resummation factor $\mathcal{R} = \frac{1}{1 - e^{-\frac{\pi\alpha}{\beta}}}$

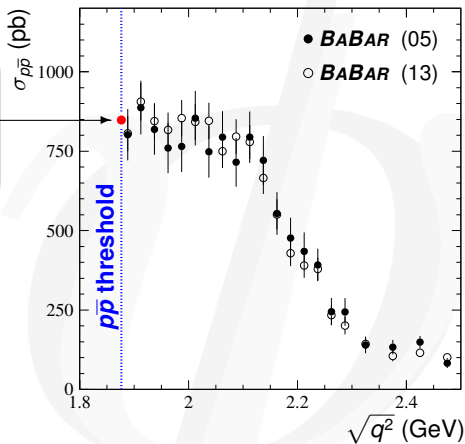
It is responsible for the multi-photon $p\bar{p}$ final state interaction, becomes ineffective few MeV above threshold and accounts also for gluon exchange.

$$\mathcal{C} = \mathcal{E} \times \mathcal{R} = \frac{\pi\alpha/\beta}{1 - \exp(-\pi\alpha/\beta)} \xrightarrow{\beta \rightarrow 0} \frac{\pi\alpha}{\beta} = \mathcal{E}$$

STEP AND PLATEAU IN *BABAR* DATA

Eur. Phys. J. A39 (2009) 315

Expected cross section with $|G_S^p(4M_p^2)| = 1$



At threshold

$$\sigma_{p\bar{p}}(4M_p^2) = \frac{\pi^2 \alpha^3}{2M_p^2} \frac{\beta_p}{\beta_p} |G_S^p(4M_p^2)|^2$$

$$\sigma_{p\bar{p}}(4M_p^2) = 0.85 |G_S^p(4M_p^2)|^2 \text{ nb}$$

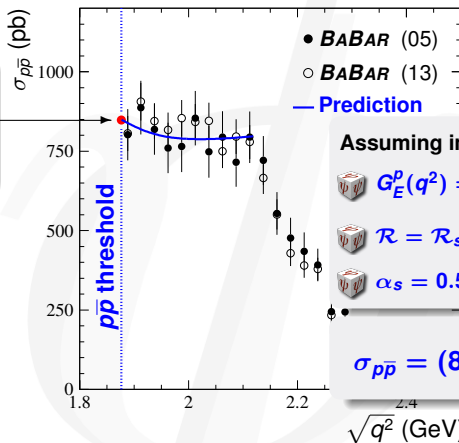


$|G_S^p(4M_p^2)| \equiv 1$
as pointlike fermion pairs!

STEP AND PLATEAU IN *BABAR* DATA

Eur. Phys. J. A39 (2009) 315

Expected cross section with $|G_S^p(4M_p^2)| = 1$



Assuming in $q^2 \leq (2.1\text{GeV})^2$:

$$G_E^p(q^2) = G_M^p(q^2) = 1$$

$$\mathcal{R} = \mathcal{R}_s = \left(1 - e^{-\frac{\pi\alpha_s}{\beta}}\right)^{-1}$$

$$\alpha_s = 0.5$$

$$\sigma_{p\bar{p}} = (850 \text{ pb}) \frac{1}{\tau} \mathcal{R}_s$$

At threshold

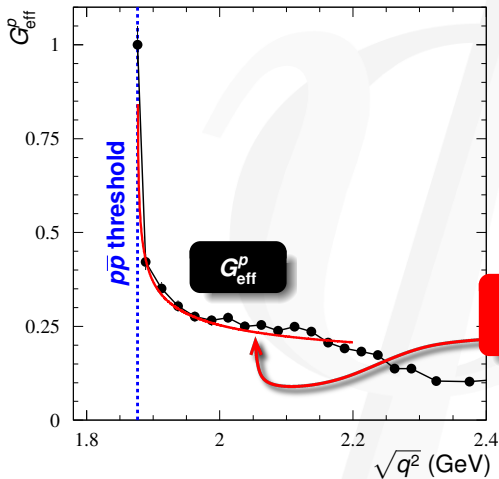
$$\sigma_{p\bar{p}}(4M_p^2) = \frac{\pi^2 \alpha^3}{2M_p^2} \frac{\beta_p}{\beta_p} |G_S^p(4M_p^2)|^2$$

$$\sigma_{p\bar{p}}(4M_p^2) = 0.85 |G_S^p(4M_p^2)|^2 \text{ nb}$$



$|G_S^p(4M_p^2)| \equiv 1$
 as pointlike fermion pairs!

G_{eff}^p , WE GET WHAT WE PUT IN...



$$G_{\text{eff}}^p(q^2) = \sqrt{\frac{1}{\mathcal{R}} \frac{\sigma_{p\bar{p}}(q^2)}{\mathcal{E} \frac{16\pi\alpha^2}{3} \frac{\beta}{4q^2} \left(1 + \frac{1}{2\tau}\right)}}$$

$$\mathcal{E} \frac{16\pi\alpha^2}{3} \frac{\beta}{4q^2} \left(1 + \frac{1}{2\tau}\right) \Big|_{\beta \rightarrow 0} \simeq \sigma_{p\bar{p}}(q^2)$$

$$G_{\text{eff}}^p(q^2) = \frac{1}{\sqrt{\mathcal{R}}}$$

$$\frac{1}{\sqrt{\mathcal{R}}} = \sqrt{1 - e^{-\frac{\pi\alpha}{\beta}}}$$

ISOTROPY AT THE $p\bar{p}$ PRODUCTION THRESHOLD

$$G_E(4M^2) = G_M(4M^2)$$



Electromagnetic current:

$$J^\mu(p_1, p_2) = \bar{U}(p_2) \left[\gamma^\mu F_1(q^2) + \frac{i\sigma^{\mu\nu} q_\nu}{2M} F_2(q^2) \right] U(p_1)$$

$$F_1 = \frac{q^2 G_E - 4M^2 G_M}{q^2 - 4M^2}$$

$$F_2 = 4M^2 \frac{G_M - G_E}{q^2 - 4M^2}$$



F_1 and F_2 “can” be analytic (pointlike limit: $F_1(q^2) = 1$ and $F_2(q^2) = 0$)



Annihilation cross section $[\tilde{G}_{E,M} \equiv G_{E,M}(4M^2)]$

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2 \beta C}{4q^2} \left[(1 + \cos^2 \theta) |G_M|^2 + \frac{1}{\tau} \sin^2 \theta |G_E|^2 \right] \xrightarrow{q^2 \rightarrow 4M^2} \frac{\alpha^2 \beta C}{16M^2} [2|\tilde{G}_M|^2]$$



Partial wave form factors

$$G_S = \frac{2\sqrt{\frac{q^2}{4M^2}} G_M + G_E}{3} \xrightarrow{q^2 \rightarrow 4M^2} \tilde{G}_M$$

$$G_D = \frac{\sqrt{\frac{q^2}{4M^2}} G_M - G_E}{3} \xrightarrow{q^2 \rightarrow 4M^2} 0$$

ANISOTROPY AT THE PRODUCTION THRESHOLD

$$G_E(4M^2) \neq G_M(4M^2)$$



Dirac and Pauli form factors F_1 and F_2 are not analytic



To preserve G_E and G_M analyticity, F_1 and F_2 must have a **simple pole** at the threshold with **opposite residues**

$$F_1 = \frac{-4M^2 \Delta \tilde{G}}{q^2 - 4M^2} + F_1^{\text{an}}$$

$$F_2 = \frac{4M^2 \Delta \tilde{G}}{q^2 - 4M^2} + F_2^{\text{an}}$$

$$\Delta \tilde{G} \equiv \tilde{G}_E - \tilde{G}_M$$

$F_{1,2}^{\text{an}}$ is the analytic part of $F_{1,2}$



Annihilation cross section

$$\frac{d\sigma}{d\Omega} \xrightarrow[q^2 \rightarrow 4M^2]{|\tilde{G}_E| \neq |\tilde{G}_M|} \frac{\alpha^2 \beta C}{8M^2} \left[|\tilde{G}_M|^2 + \text{Re}(\Delta \tilde{G} \tilde{G}_M^*) \sin^2 \theta \right]$$

Assuming
 $|\Delta \tilde{G}| \ll |\tilde{G}_M|$

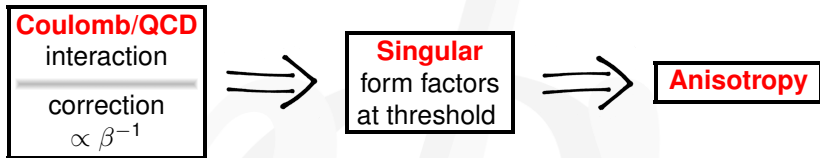


Partial wave form factors

$$G_S = \frac{2\sqrt{\frac{q^2}{4M^2}} G_M + G_E}{3} \xrightarrow[q^2 \rightarrow 4M^2]{} \tilde{G}_M + \frac{\Delta \tilde{G}}{3}$$

$$G_D = \frac{\sqrt{\frac{q^2}{4M^2}} G_M - G_E}{3} \xrightarrow[q^2 \rightarrow 4M^2]{} -\frac{\Delta \tilde{G}}{3}$$

SOURCES OF ANISOTROPY



Only Coulomb

Dmitriev, Milstein, PLB722 (13) 83

$$\tilde{G}_D \sim -\frac{\alpha^2}{8}$$

Very small
but not vanishing!

QCD Coulomb like

Brodsky, Hoang, Kuhn, Teubner,
PB359 (95) 355

Large effect for
heavy quarks

Anisotropy $\propto \beta^n$
No effect at threshold!

MEASURING ANISOTROPY AT THRESHOLD

$$e^+ e^- \rightarrow p \bar{p}$$

SND

CMD3

Very difficult

Efficiency drops with proton antiproton velocity

$$p \bar{p} \rightarrow e^+ e^-$$

PANDA

Very difficult

Normalization (Coulomb corrections...)

$$e^+ e^- \rightarrow p \bar{p} \gamma$$

BABAR

BESIII

Difficult

ISR technique: not enough statistics

$$e^+ e^- \rightarrow H_B \bar{H}_B$$

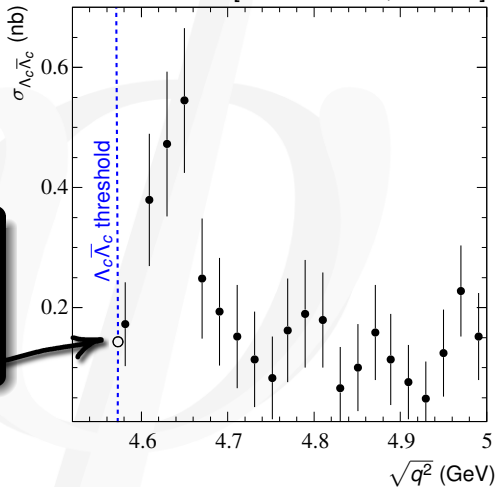
BESIII

Feasible with heavy baryons

The weak decay allows detection at threshold and polarization measurements (BESIII has...)

THE $e^+e^- \rightarrow \Lambda_c \bar{\Lambda}_c$ CROSS SECTION

[Belle PRL101, 172001]



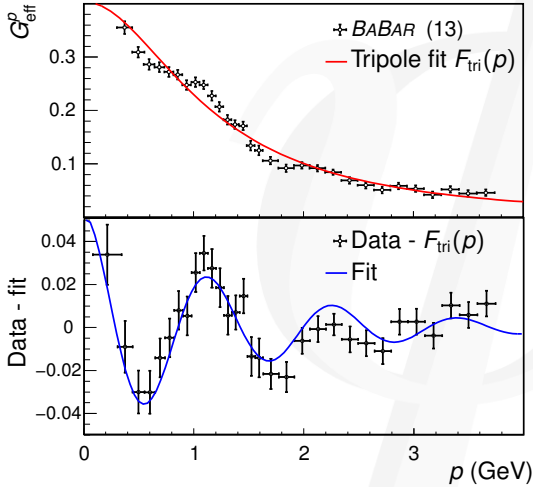
Cross section
at threshold
assuming

$$|G^{\Lambda_c}(4M_{\Lambda_c}^2)| = 1$$

BESIII is working on that
New results are coming soon

PERIODIC INTERFERENCE NEAR THRESHOLD

A. BIANCONI, E. TOMASI-GUSTAFSSON, PHYS. REV. LETT. 114, 232301



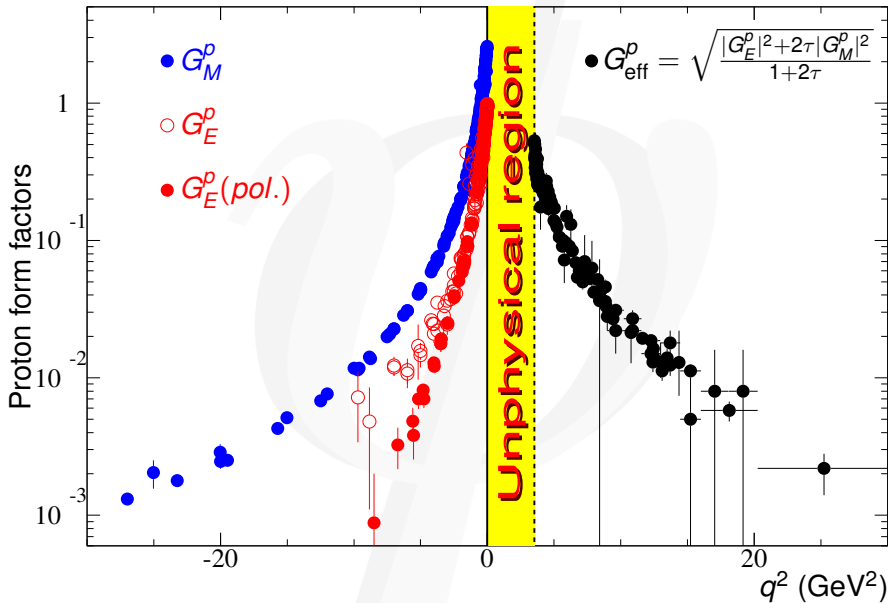
p is the momentum of the proton in the anti-proton rest frame.

The periodical behavior suggests an interference due to a rescattering of proton and antiproton at low kinetic energy and separation ~ 1 fm.

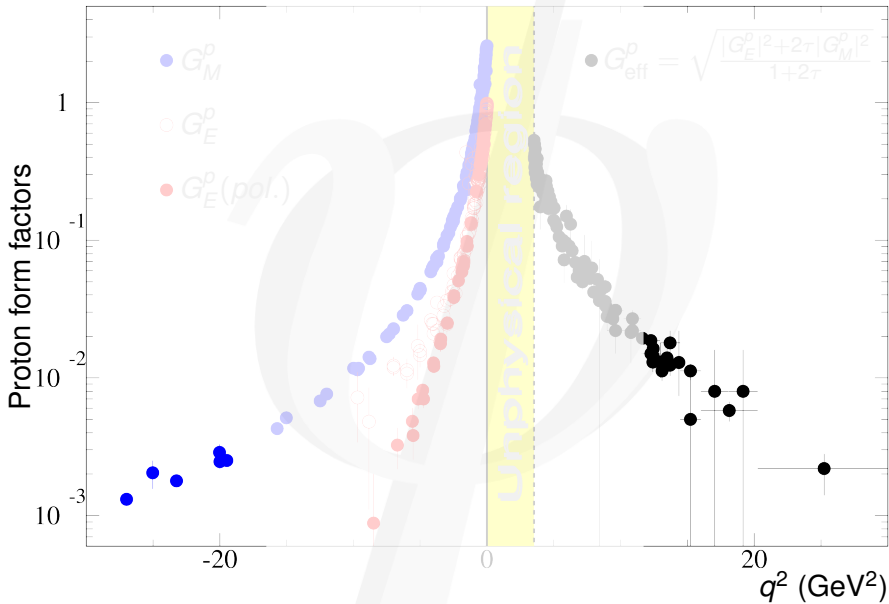
Proton and antiproton interact when they are almost phenomenological.

Unitarity implies a large imaginary part of form factors.

THE ASYMPTOTIC REGIONS₁

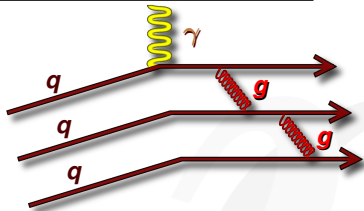



THE ASYMPTOTIC REGIONS₁




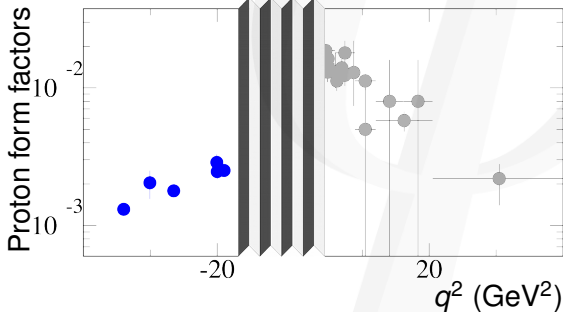
THE ASYMPTOTIC REGIONS₁

Space-like dimensional scaling




 pQCD: as $q^2 \rightarrow -\infty$, asymptotic behaviors of F_1 and F_2 , and G_E and G_M must follow counting rules

 Valence quarks exchange gluons to distribute the momentum transfer q



 Dirac and Pauli form factors

$$F_i(q^2) \underset{q^2 \rightarrow -\infty}{\sim} (-q^2)^{-1-i}$$

 Sachs form factors

$$G_{E,M}(q^2) \underset{q^2 \rightarrow -\infty}{\sim} (-q^2)^{-2}$$

$$\frac{G_E(q^2)}{G_M(q^2)} \underset{q^2 \rightarrow -\infty}{\sim} \text{const.}$$

THE ASYMPTOTIC REGIONS₁

Time-like asymptotic behavior

Phragmén Lindelöf theorem:

If a function $f(z) \rightarrow a$ as $z \rightarrow \infty$ along a straight line, and $f(z) \rightarrow b$ as $z \rightarrow \infty$ along another straight line, and $f(z)$ is regular and bounded in the angle between, then $a = b$ and $f(z) \rightarrow a$ uniformly in this angle.

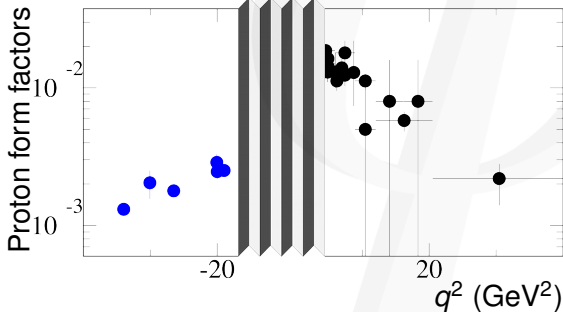


$$\lim_{q^2 \rightarrow -\infty} G_{E,M}(q^2) = \lim_{q^2 \rightarrow +\infty} G_{E,M}(q^2)$$

space-like time-like

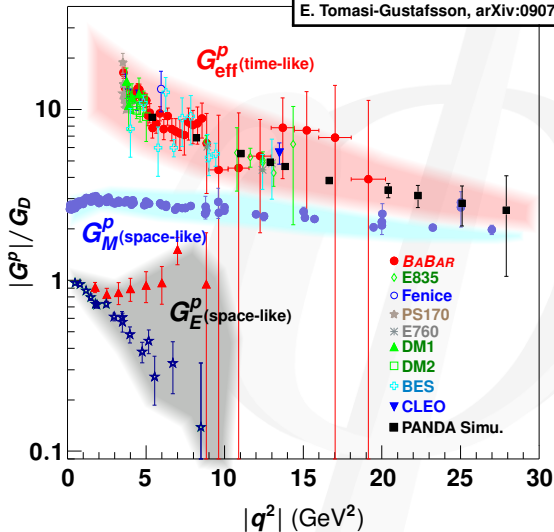


$$G_{E,M} \sim (q^2)^{-2} \quad \text{real}$$



THE ASYMPTOTIC REGIONS₂

E. Tomasi-Gustafsson and M. P. Rekalo, PLB504,291
E. Tomasi-Gustafsson, arXiv:0907.4442



— pQCD —

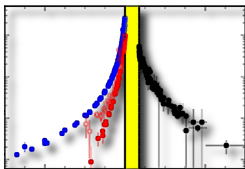
$$G_{\text{eff}}^p(q^2) \underset{q^2 \rightarrow \infty}{\sim} G_M^p(q^2)$$

— Phragmèn Lindelöf —

$$\lim_{q^2 \rightarrow \infty} \frac{G_{\text{eff}}^p(q^2)}{G_M^p(-q^2)} = 1$$







— — —

CONCLUSIONS








Global models for proton and neutron, electric and magnetic form factors must be encouraged.

They can help in understanding:

-  the threshold behavior
-  the proton radius
-  the presence of zeros
-  the asymptotic behavior
-  the unphysical region
-  ...



To measure:

-  zero of G_E^p in space-like region
-  moduli of G_E and G_M in time-like region
-  complex structure of form factors (polarization)
-  unphysical time-like form factors ($p\bar{p} \rightarrow \pi^0 e^+ e^-$)
-  ...

EXPERIMENTS: NOW AND FUTURE

Space-like
region



- G_E^n at $-q^2 = 1.5 \text{ GeV}^2$ (Pol. ^3He)
- G_E^p and G_M^p for $-q^2 \leq 2.0 \text{ GeV}^2$

Jefferson Lab

- [Hall A] G_E^p/G_M^p up to 14 GeV^2
- [Hall A] G_M^n (ratio) up to 18 GeV^2
- [Hall A] G_E^n/G_M^n up to 10.2 GeV^2
- [Hall B] G_M^n (deuterium) up to 14 GeV^2
- [Hall C] G_E^n up to 7 GeV^2

Time-like
region



at VEPP-2000
 e^+e^- collider



$$|G_{\text{eff}}^p|, |G_{\text{eff}}^n| \text{ (scan)}$$

$$q^2 \leq (4 \text{ GeV})^2$$

BESIII



at BEPCII
 e^+e^- collider

$$|G_E^p|, |G_M^p|, |G_{\text{eff}}^n| \text{ (scan and ISR)}$$

$$q^2 \leq (3.5 \text{ GeV})^2$$

panda

at FAIR
 $p\bar{p}$ collider

$$|G_E^p|, |G_M^p|, G_E^p/G_M^p \text{ phase (}\bar{p} \text{ polarization)}$$

$$(2.4 \text{ GeV})^2 \leq q^2 \leq (3.7 \text{ GeV})^2$$

Belle II

?

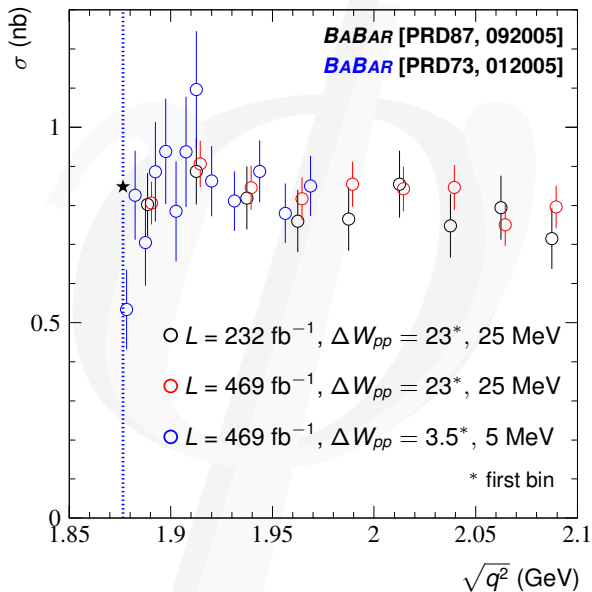
at SuperKEKB
 e^+e^- collider

$$|G_E^p|, |G_M^p|, \text{ (ISR)}$$

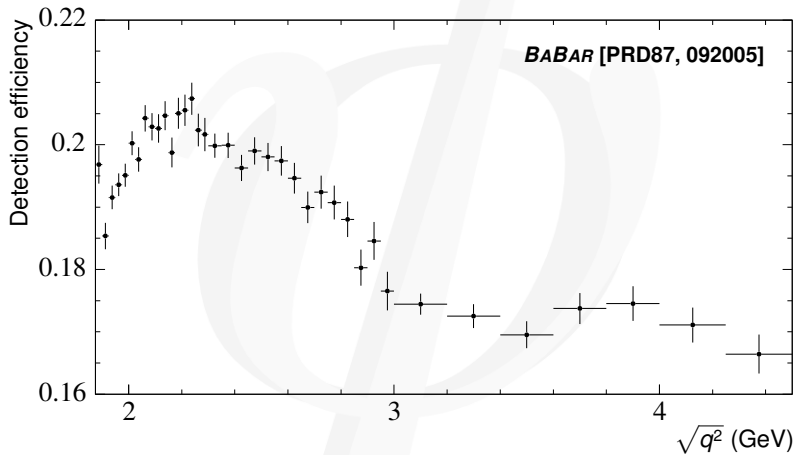
$$q^2 \leq (4.5 \text{ GeV})^2$$

Additional slides

THE THRESHOLD REGION₃



THE THRESHOLD REGION₃



AN INTERESTING MODEL

E. A. Kuraev, E. Tomasi-Gustafsson, A. Dbeyssi PLB712 240

Assumption

Pauli principle pulls away from the internal region of strong chromo-electromagnetic field quarks of same flavor because the color quantum number does not play any role (stochastic averaging).

Outer spatial region

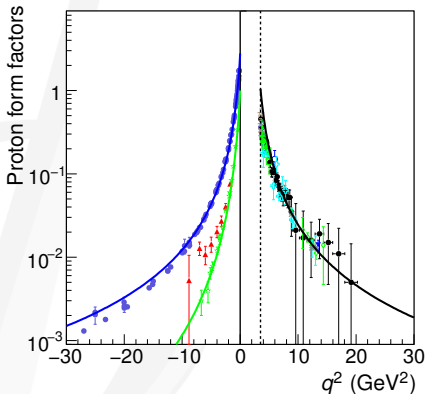
$$|p\rangle = \epsilon^{ijk} |u_i u_j d_k\rangle$$

charge = 1

Central region

$$|p\rangle \neq \epsilon^{ijk} |u_i u_j d_k\rangle$$

charge = 0



Counting rule on the vector part of interaction

$$G_M^p(q^2) = \mu_p G_D(q^2)$$

space-like

A screening effect from the central region provides an additional suppression for the electric form factor

$$G_E^p(q^2) = \frac{G_D(q^2)}{1 - q^2/q_1^2}$$

time-like

$$G_M^p(q^2) = \frac{\theta(q^2 - 4M_p^2)}{[1 + (q^2 - 4M_p^2)^2/q_2^2]^2}$$

$$G_E^p(q^2) = \frac{G_M^p(q^2) \theta(q^2 - 4M_p^2)}{1 + (q^2 - 4M_p^2)^2/q_2^2}$$

PARTIAL WAVE FORM FACTORS



$$\begin{cases} P_\gamma = -1 \\ J_\gamma = 1 \end{cases} \quad \begin{cases} P_{p\bar{p}} = (-1)^{L+1} \\ S_{p\bar{p}} = 0, 1 \end{cases}$$



$$\begin{cases} L_{p\bar{p}} = 0, 2 \\ S_{p\bar{p}} = 1 \end{cases}$$

~~$$\begin{cases} L_{p\bar{p}} = 1 \\ S_{p\bar{p}} = 0 \end{cases}$$~~

Partial wave form factors

$$G_S^p = \frac{1}{3} \left(2G_M^p \sqrt{\frac{q^2}{4M_p^2}} + G_E^p \right)$$

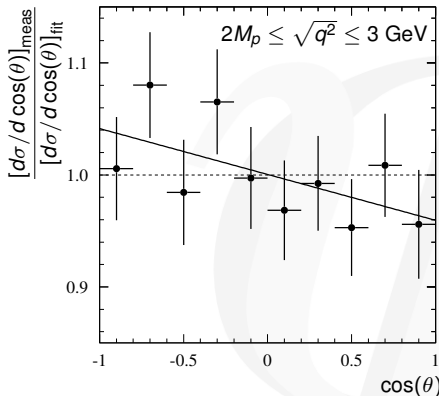
$$G_D^p = \frac{1}{3} \left(G_M^p \sqrt{\frac{q^2}{4M_p^2}} - G_E^p \right)$$

Cross section

$$\sigma(q^2) = 2\pi\alpha^2\beta \frac{4M_p^2}{(q^2)^2} \left[C |G_S^p(q^2)|^2 + 2|G_D^p(q^2)|^2 \right]$$

$\gamma\gamma$ EXCHANGE FROM $e^+e^- \rightarrow p\bar{p}\gamma$ **BABAR** 2013 DATA

E. Tomasi-Gustafsson, E.A. Kuraev, S. Bakmaev, SP, Phys. Lett. B659 (2008) 197
Phys. Rev. D87 (2013) 092005



Integrated over the $p\bar{p}$ -CM energy from threshold up to 3 GeV

The MC-fit assumes
one-photon exchange

Slope = $-0.041 \pm 0.026 \pm 0.005$

Integral asymmetry

$$\langle \mathcal{A} \rangle_{\cos \theta_p} = \frac{\sigma(\cos \theta_p > 0) - \sigma(\cos \theta_p < 0)}{\sigma(\cos \theta_p > 0) + \sigma(\cos \theta_p < 0)} = -0.025 \pm 0.014 \pm 0.003$$

$\sigma(\cos \theta_p \geq 0)$ is the cross section integrated with $\sqrt{q^2} \leq 3 \text{ GeV}$ and $\cos \theta_p \geq 0$