Relative phase of Λ form factors

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Electromagnetic Structure of Strange Baryons

GSI Helmholtzzentrum für Schwerionenforschung GmbH

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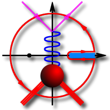
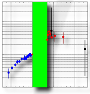
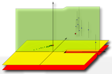
Agenda



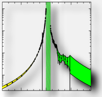
Baryon form factors and dispersion relations



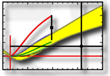
Space-like and time-like data on G_E/G_M



Space-like and time-like G_E/G_M via DR's



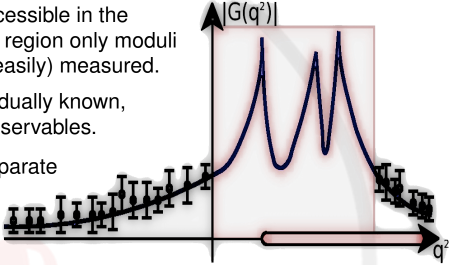
Asymptotic G_M from a DR sum rule



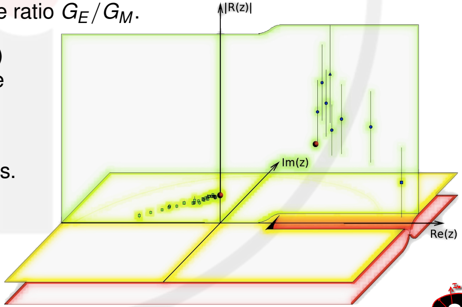
Hints for the ratio of Λ form factors

About proton form factors

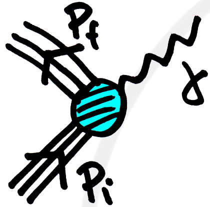
- Proton form factors are completely accessible in the space-like region while in the time-like region only moduli above the physical threshold can be (easily) measured.
- In the space-like region they are individually known, especially by means of polarization observables.
- Only recently, attempts to measure separate value of moduli in the time-like region have become stronger.



- So far, the better known quantity is the ratio G_E/G_M.
- Using dispersion relations (analyticity) space-like and time-like values can be exploited to extract information on phase and asymptotic behavior.
- Analyticity imposes serious constraints. A space-like zero for the ratio G_E/G_M does require an asymptotic phase of 180 degrees.



Proton-photon vertex



Nucleon electromagnetic four-current ($q = p_f - p_i$)

$$\langle P_f | J^\mu_{\text{EM}}(0) | P_i \rangle = e\,\bar{u}(p_f)\left[\gamma^\mu F_1(q^2) + \frac{i\sigma^{\mu\nu} q_\nu}{2M_p} F_2(q^2)\right] u(p_i)$$

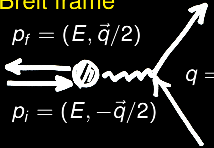
$F_1(q^2)$ and $F_2(q^2)$ are the Dirac and Pauli form factors

$F_1(0) = \mathcal{Q}_p$ $\qquad$ $F_2(0) = \kappa_p$

$\mathcal{Q}_p$ = electric charge $\qquad$ κ_p = anomalous magnetic moment

Breit frame

$p_f = (E, \vec{q}/2)$



$q = (0, \vec{q})$

$p_i = (E, -\vec{q}/2)$

$$\langle P_f | J^\mu_{\text{EM}}(0) | P_i \rangle \equiv J^\mu_{\text{EM}} = \left(J^0_{\text{EM}}, \vec{J}_{\text{EM}}\right)$$

- $J^0_{\text{EM}} = e\left(F_1(q^2) + \dfrac{q^2}{4M_p^2} F_2(q^2)\right)$
- $\vec{J}_{\text{EM}} = e\,\bar{u}(p_f)\vec{\gamma}u(p_i)\left(F_1(q^2) + F_2(q^2)\right)$

Sachs form factors

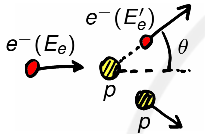
- $G_E(q^2) = F_1(q^2) + \dfrac{q^2}{4M_p^2} F_2(q^2)$
- $G_M(q^2) = F_1(q^2) + F_2(q^2)$

Normalizations

- $G_E(0) = \mathcal{Q}_p$
- $G_M(0) = \mu_p = \kappa_p + \mathcal{Q}_p$

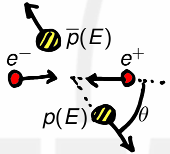
μ_p = total magnetic moment

Cross sections and Coulomb correction



Elastic scattering cross section (Rosenbluth)

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2 E'_e \cos^2\left(\frac{\theta}{2}\right)}{4E_e^3 \sin^4\left(\frac{\theta}{2}\right)} \left[G_E^2 - \tau\left(1+2(1-\tau)\tan^2\left(\frac{\theta}{2}\right)\right) G_M^2\right] \frac{1}{1-\tau}$$



Annihilation cross section

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2 \beta\, \mathcal{C}}{16E^2} \left[\left(1+\cos^2(\theta)\right)|G_M|^2 + \frac{1}{\tau}\sin^2(\theta)\,|G_E|^2\right]$$

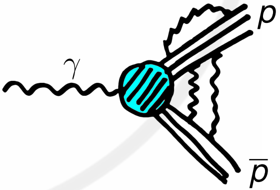
$$\tau = E^2/M_p^2$$

$$\beta = \sqrt{1-1/\tau}$$

Coulomb correction

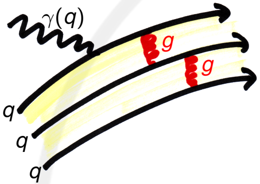
$$\mathcal{C} = \frac{\pi\alpha/\beta}{1-e^{-\pi\alpha/\beta}}$$

- $p\overline{p}$ Coulomb interaction as FSI
- Only S-wave



pQCD asymptotic behavior
Space-like region

V.A. Matveev, R.M. Muradian, A.N. Tavkhelidze, LNC7 (1973) 719
S. J. Brodsky, G. R. Farrar, PRL31 (1973) 1153
M. V. Galynsky, E. A. Kuraev JETPL96 (2012) 6



- **pQCD**: as $q^2 \to -\infty$, F_1, F_2, G_E, G_M follow power laws driven by counting rules
- Valence quarks exchange gluons to distribute the photon momentum transfer q

Non-helicity-flip current $J^{\lambda,\lambda}(q^2)$

- $J^{\lambda,\lambda}(q^2) \propto G_M(q^2)$
- Two gluon propagators
- $G_M(q^2) \underset{q^2\to-\infty}{\sim} \left(q^2\right)^{-2}$

Helicity-flip current $J^{\lambda,-\lambda}(q^2)$

- $J^{\lambda,-\lambda}(q^2) \propto G_E(q^2)/\sqrt{-q^2}$
- [Two gluon propagators]$/\sqrt{-q^2}$
- $G_E(q^2) \underset{q^2\to-\infty}{\sim} \left(q^2\right)^{-2}$

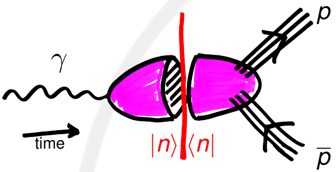
Dirac and Pauli form factors

- $F_1(q^2) \underset{q^2\to-\infty}{\sim} \left(q^2\right)^{-2}$
- $F_2(q^2) \underset{q^2\to-\infty}{\sim} \left(q^2\right)^{-3}$

Ratio of Sachs form factors

- $\dfrac{G_E(q^2)}{G_M(q^2)} \underset{q^2\to-\infty}{\sim}$ constant

Nucleon form factors
Time-like region ($q^2 > 0$)



- Crossing symmetry:

$$\langle P(p')|j^\mu|P(p)\rangle \to \langle \overline{P}(p')P(p)|j^\mu|0\rangle$$

- Form factors are complex functions of q^2

Optical theorem

$$\mathrm{Im}\langle \overline{P}(p')P(p)|j^\mu|0\rangle \sim \sum_n \langle \overline{P}(p')P(p)|j^\mu|n\rangle\langle n|j^\mu|0\rangle \implies \begin{cases} \mathrm{Im}F_{1,2} \neq 0 \\ \text{for } q^2 > 4M_\pi^2 \end{cases}$$

$|n\rangle$ are on-shell intermediate states: 2π, 3π, 4π, ...

Time-like asymptotic behavior

Phragmèn Lindelöf theorem

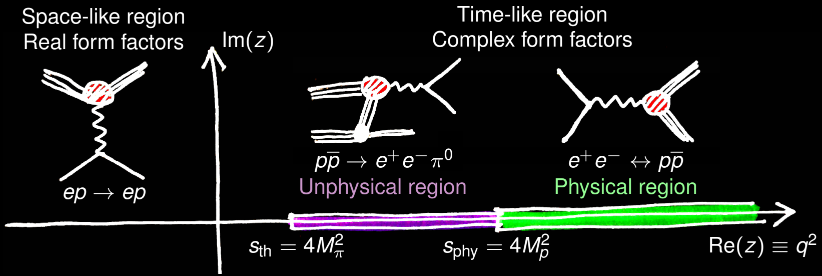
If $\boldsymbol{f(z)} \longrightarrow \boldsymbol{a}$ as $z \to \infty$ along a straight line, and $\boldsymbol{f(z)} \longrightarrow \boldsymbol{b}$ as $z \to \infty$ along another straight line, and $f(z)$ is regular and bounded in the angle between, then $\boldsymbol{a} = \boldsymbol{b}$ and $\boldsymbol{f(z)} \longrightarrow \boldsymbol{a}$ **uniformly in this angle**.

- $$\underbrace{\lim_{q^2\to-\infty} G_{E,M}(q^2)}_{\text{space-like}} = \underbrace{\lim_{q^2\to+\infty} G_{E,M}(q^2)}_{\text{time-like}}$$

- $$G_{E,M} \underset{q^2\to+\infty}{\sim} (q^2)^{-2}$$ **Must be real**

Analyticity of form factors

q^2-complex plane

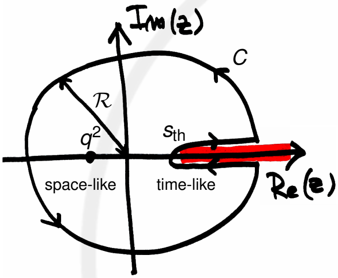


Only the real axis of the q^2-complex plane is experimentally accessible

Space-like region $q^2 < 0$	Time-like region* $s_{th} < q^2 \leq s_{phy}$	Time-like region $q^2 \geq s_{phy}$	
$ep \to ep$	$p\bar{p} \to e^+e^-\pi^0$	$e^+e^- \leftrightarrow p\bar{p}$	$e^+e^- \leftrightarrow p\bar{p}$ (pol.)
G_E, G_M	$\lvert G_E\rvert, \lvert G_M\rvert$	$\lvert G_E\rvert, \lvert G_M\rvert$	$\lvert G_E\rvert, \lvert G_M\rvert, \arg(G_E/G_M)$

*C. Adamuscin, E.A. Kuraev, E. Tomasi-Gustafsson, F. Maas PRC75, 045205
E. A. Kuraev et al., JETP115, 93
G. I. Gakh, E. Tomasi-Gustafsson, A. Dbeyssi, A.G. Gakh PRC86, 025204

Dispersion relations



- The form factors are analytic on the q^2-plane with a multiple cut ($s_{\rm th} = 4M_\pi^2, \infty$)
- Dispersion relation for the imaginary part ($q^2 < 0$)

$$G(q^2) = \lim_{\mathcal{R}\to\infty} \frac{1}{2\pi i}\oint_C \frac{G(z)dz}{z - q^2} = \frac{1}{\pi}\int_{s_{\rm th}}^{\infty} \frac{\mathrm{Im}G(s)ds}{s - q^2}$$

- Dispersion relation for the logarithm ($q^2 < 0$)
  B.V. Geshkenbein, Yad. Fiz. 9 (1969) 1232.

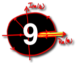
$$\ln G(q^2) = \frac{\sqrt{s_{\rm th} - q^2}}{\pi}\int_{s_{\rm th}}^{\infty} \frac{\ln|G(s)|ds}{(s - q^2)\sqrt{s - s_{\rm th}}}$$

Experimental inputs

- Space-like data on the real values of form factors from: $ep \to ep$ and $e^\uparrow p \to e^- p^\uparrow$, with polarization
- Time-like data on form factor moduli from: $e^+e^- \leftrightarrow p\bar{p}$
- Time-like data on G_E/G_M phase from: $e^+e^- \leftrightarrow p^\uparrow \bar{p}$ (pol.)

Theoretical ingredients

- Analyticity $\Rightarrow$ convergence relations
- Normalization and threshold values
- Asymptotic behavior $\Rightarrow$ super-convergence relations



Advantages and drawbacks of dispersive approaches

Advantages

- DR's are based on unitarity and analyticity $\Rightarrow$ model-independent approach
- DR's relate data from different processes in different energy regions

$$\left[\begin{array}{c}\textbf{space-like}\\ \textbf{form factor}\\ ep \to ep\end{array}\right] = \int_{s_{\rm th}}^{\infty} \left[\begin{array}{c}\textbf{Im(form factor) or ln}|\textbf{form factor}|\\ \textbf{over the time-like cut } (s_{\rm th}, \infty)\\ e^+e^- \to p\bar{p} + \textbf{theory}\end{array}\right]$$

- Normalizations and theoretical constraints can be directly implemented
- Form factors can be computed in the whole q^2-complex plane

Drawbacks

- Very long-range integration

Remedy #1
pQCD power laws

Remedy #2
Subtracted DR's

- No data in the unphysical region, crucial in dispersive analyses

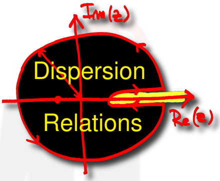
A DISPERSIVE APPROACH FOR GE/GM

Dispersive approach for the ratio $R = \mu_p G_E / G_M$

We start from the imaginary part of the ratio $R(q^2)$, written in the most general and model-independent way as

$$I(q^2) \equiv \mathrm{Im}[R(q^2)] = \text{series of orthogonal polynomials}$$

Theoretical constraints can be applied directly on this function $I(q^2)$



The function $R(q^2)$ is reconstructed in time and space-like regions

Additional theoretical conditions and the experimental constraints can be imposed on the obtained analytic expression of $R(q^2)$

Parametrization for G_E/G_M

The imaginary part of $R(q^2)$ is parametrized by two series of orthogonal polynomials

$$\mathrm{Im}\left[R(q^2)\right] \equiv I(q^2) = \begin{cases} \sum_i C_i T_i(x) & x = \frac{2q^2 - s_{\rm phy} - s_{\rm th}}{s_{\rm phy} - s_{\rm th}} \quad s_{\rm th} \leq q^2 \leq s_{\rm phy} \\ \sum_j D_j T_j(x') & x' = \frac{2s_{\rm phy}}{q^2} - 1 \quad q^2 > s_{\rm phy} \end{cases}$$

Theoretical conditions on $\mathrm{Im}\left[R(q^2)\right]$

- $R(4M_\pi^2)$ is real $\Longrightarrow I(4M_\pi^2) = 0$
- $R(4M_p^2)$ is real $\Longrightarrow I(4M_p^2) = 0$
- $R(\infty)$ is real $\Longrightarrow I(\infty) = 0$

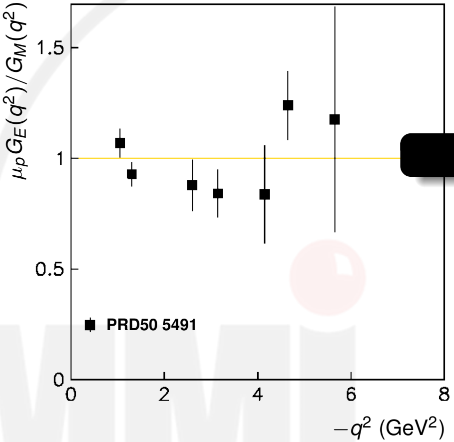
Theoretical conditions on $R(q^2)$

- Continuity at $q^2 = 4M_\pi^2$
- $R(4M_p^2)$ is real and $\mathrm{Re}\left[R(4M_p^2)\right] = \mu_p$

Experimental conditions on $R(q^2)$ and $|R(q^2)|$

- Space-like region ($q^2 < 0$) data for R from JLab and MIT-Bates
- Time-like region ($q^2 \geq 4M_p^2$) data for $|R|$ from FENICE+DM2, *BaBar* , and E835

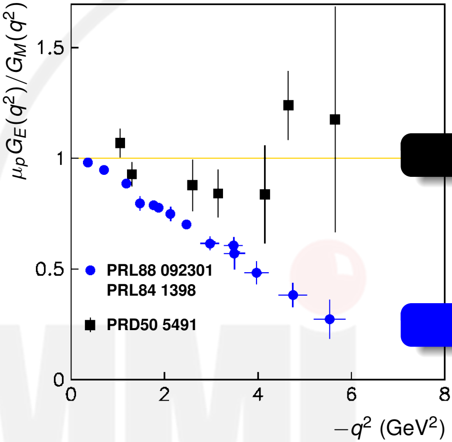
Space-like data on G_E/G_M



Rosenbluth

Radiative corrections of polarization technique ≪ Radiative corrections in Rosenbluth method

Space-like data on G_E/G_M

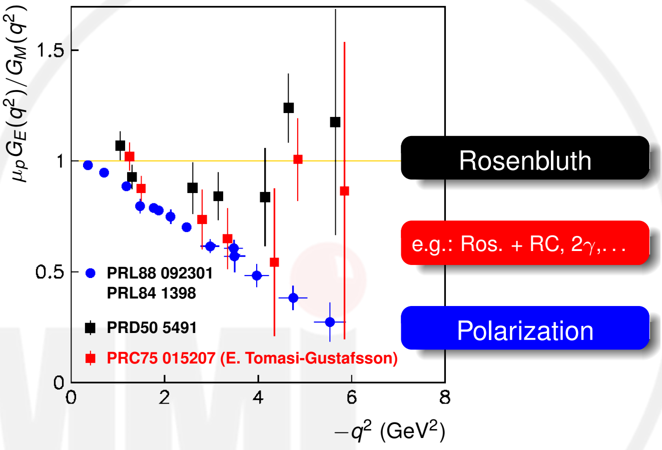


Rosenbluth

Polarization

Radiative corrections of polarization technique $\ll$ Radiative corrections in Rosenbluth method

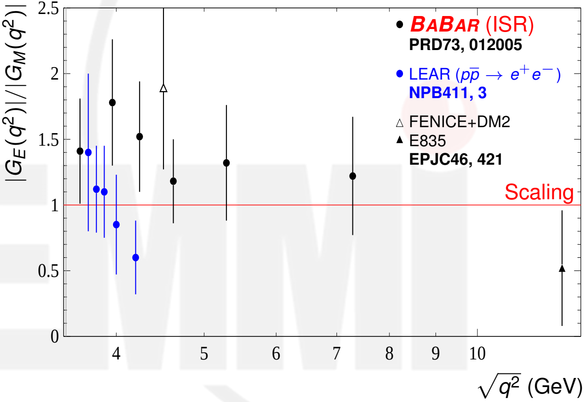
Space-like data on G_E/G_M



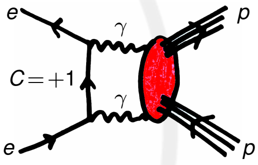
Radiative corrections of polarization technique $\ll$ Radiative corrections in Rosenbluth method

Time-like data on $|G_E/G_M|$

$$\frac{d\sigma}{d\cos(\theta)} = \frac{\pi\alpha^2\beta C}{2q^2}|G_M|^2\left[(1+\cos^2(\theta)) + \frac{4M_p^2}{q^2}\sin^2(\theta)\left|\frac{G_E}{G_M}\right|^2\right]$$



Two-photon exchange



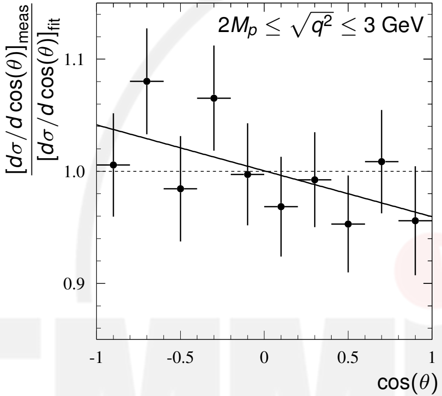
$\gamma\gamma$ exchange interferes with the Born term

Asymmetry in angular distributions

[E. Tomasi-Gustafsson and Q. H. Zhou]

Time-like two-photon contribution?

Phys. Lett. B659 (2008) 197
arXiv:1302.0055



Integrated over the $p\bar{p}$ - CM energy from threshold up to 3 GeV

The MC-fit assumes
one-photon exchange

Slope $= -0.041 \pm 0.026 \pm 0.005$

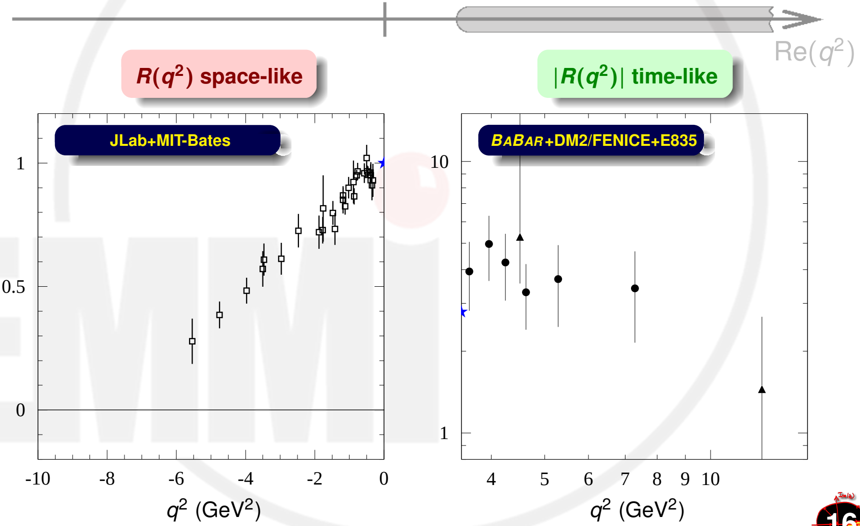
Integral asymmetry

$$\langle \mathcal{A} \rangle_{\cos\theta_p} = \frac{\sigma(\cos\theta_p > 0) - \sigma(\cos\theta_p < 0)}{\sigma(\cos\theta_p > 0) + \sigma(\cos\theta_p < 0)} = -0.025 \pm 0.014 \pm 0.003$$

$\sigma(\cos\theta_p \gtrless 0)$ is the cross section integrated with $\sqrt{q^2} \leq 3$ GeV and $\cos\theta_p \gtrless 0$

The result for $R = \mu_p G_E / G_M$

$$R(q^2) = R(0) + \frac{q^2}{\pi} \int_{4M_\pi^2}^{\infty} \frac{\mathrm{Im}\,[R(s)]}{s(s-q^2)} ds$$



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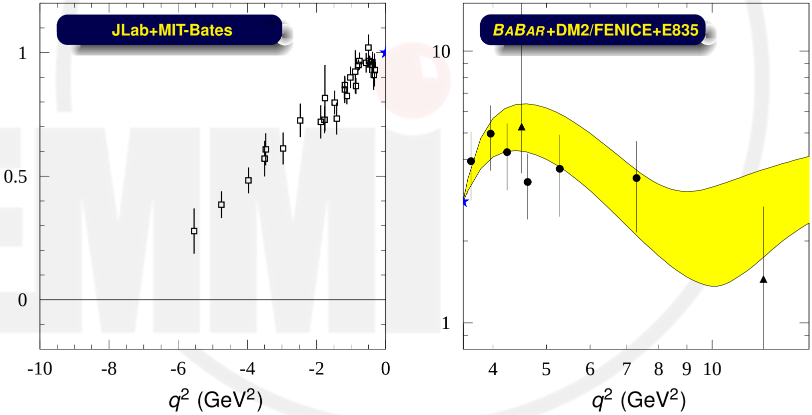
$R(q^2)$ space-like **$|R(q^2)|$ time-like**



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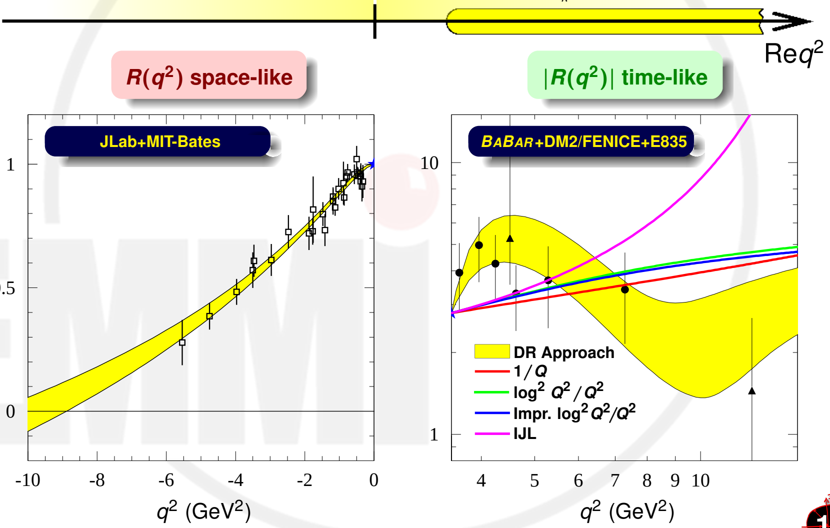
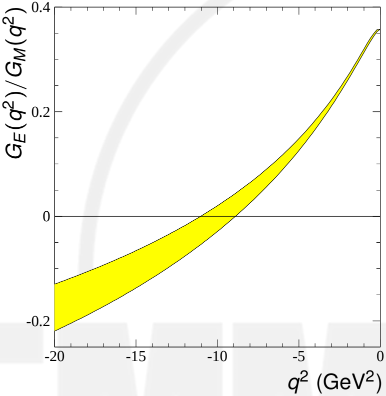
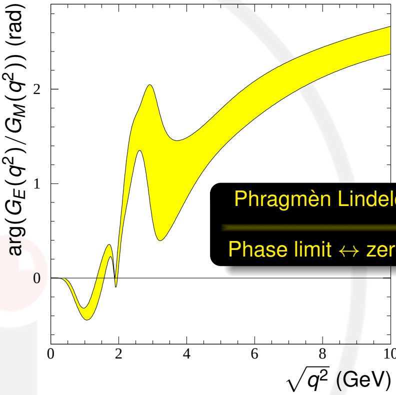


Space-like zero and phase





Phragmèn Lindelöf

Phase limit $\leftrightarrow$ zeros

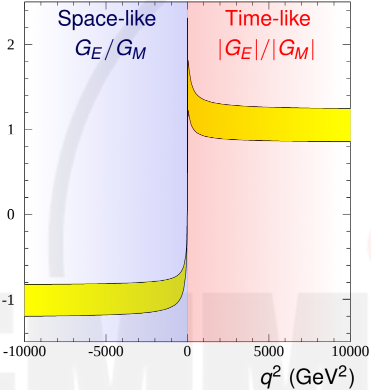
Space-like zero

$t_0^{BABAR} = (-10 \pm 1)$ GeV^2

Phase from dispersion relations

$$\phi(q^2) = -\frac{\sqrt{q^2 - s_{th}}}{\pi} \, \mathrm{Pr}\int_{s_{th}}^{\infty} \frac{\ln|R(s)|\,ds}{\sqrt{s - s_{th}}\,(s - q^2)}$$

Asymptotic G_E^p/G_M^p



Real asymptotic values for G_E/G_M

- $$\frac{G_E}{G_M} \underset{|q^2|\to\infty}{\longrightarrow} -1.0 \pm 0.2$$

Asymptotic behaviour of F_2/F_1

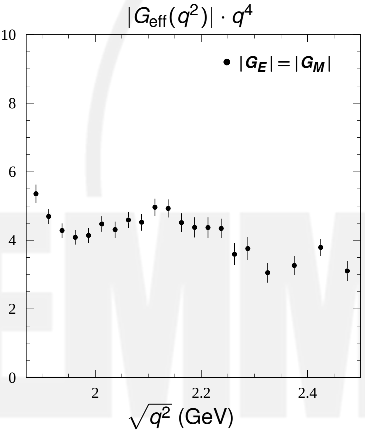
- $$\left|\frac{q^2}{4M_N^2}\frac{F_2}{F_1}\right| \underset{|q^2|\to\infty}{\longrightarrow} \left|\frac{G_E}{G_M} - 1\right| = 2.0 \pm 0.2$$

pQCD prediction

$$\left|\frac{G_E(q^2)}{G_M(q^2)}\right| \underset{|q^2|\to\infty}{\longrightarrow} 1$$

$|G_E^p|$ and $|G_M^p|$ from $p\bar{p}$ cross section and dispersion relations

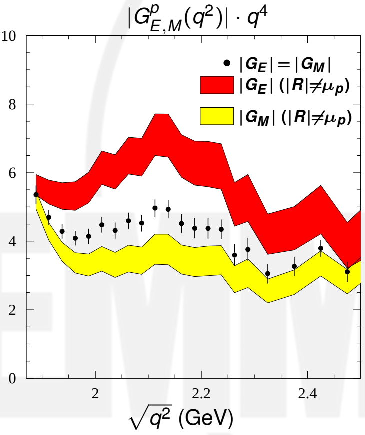
EPJA32, 421



$$|G_{\rm eff}(q^2)|^2 = \frac{\sigma_{p\bar{p}}(q^2)}{\frac{4\pi\alpha^2\beta\mathcal{C}}{3s}} \left(1 + \frac{1}{2\tau}\right)^{-1}$$

- Usually what is extracted from the cross section $\sigma(e^+e^- \to p\bar{p})$ is the effective time-like form factor $|G^p_{\rm eff}|$ obtained assuming $|G_E^p| = |G_M^p|$ i.e. $|R| = \mu_p$

EPJA32, 421

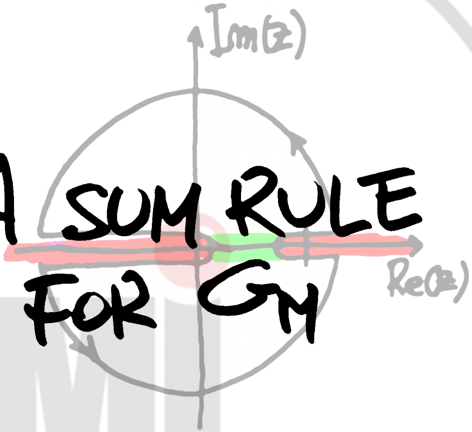


$$|G_M(q^2)|^2 = \frac{\sigma_{p\overline{p}}(q^2)}{\frac{4\pi\alpha^2\beta C}{3s}} \left(1 + \frac{|R(q^2)|}{2\mu_p\tau}\right)^{-1}$$

- Usually what is extracted from the cross section $\sigma(e^+e^- \to p\overline{p})$ is the effective time-like form factor $|G^p_{\rm eff}|$ obtained assuming $|G^p_E| = |G^p_M|$ i.e. $|R| = \mu_p$

- Using our parametrization for R and the ***BaBar*** data on $\sigma(e^+e^- \to p\overline{p})$, $|G^p_E|$ and $|G^p_M|$ may be disentangled

A SUM RULE FOR G_M

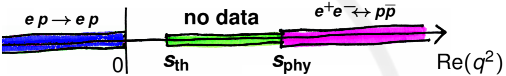


Dispersion relations and sum rules

Geshkenbeĭn, Ioffe, Shifman Yad. Fiz. 20, 128 (1974)

- DR's connect space and time values of a form factor $G(q^2)$

$$G(q^2) = \frac{1}{\pi}\int_{s_{th}}^{\infty} \frac{\mathrm{Im}G(s)ds}{s-q^2}$$



Drawbacks

- The imaginary part is not experimentally accessible
- There are no data in the unphysical region $[s_{th}, s_{phy}]$
- We need to know the asymptotic behavior

- They applied the DR for the imaginary part to the function

$$\phi(z) = f(z)\frac{\ln G(z)}{z\sqrt{s_{th}-z}} \quad \text{with} \quad \int_0^{s_{phy}} f^2(z)dz << 1$$

Advantages

- The DR integral contains the modulus $|G(s)|$
- The unphysical region contribution is suppressed

Drawback

- Zeros of $G(z)$ are poles for $\phi(z)$

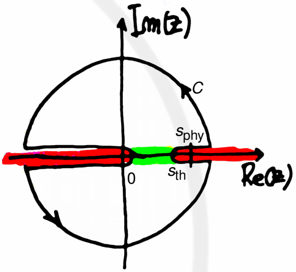
Attenuated DR and sum rule

Assuming $G(q^2) \neq 0$ and using the Cauchy theorem, we have the new DR

$$\oint_C \phi(z)dz = 0$$

$$\Downarrow$$

$$\underbrace{-\int_{-\infty}^{0} \frac{\mathrm{Im}[f(t)] \ln G(t)}{t\sqrt{s_{\mathrm{th}} - t}} dt}_{\text{Space-like}} = \underbrace{\int_{s_{\mathrm{th}}}^{\infty} \frac{f(s) \ln |G(s)|}{s\sqrt{s - s_{\mathrm{th}}}} ds}_{\text{Time-like}}$$



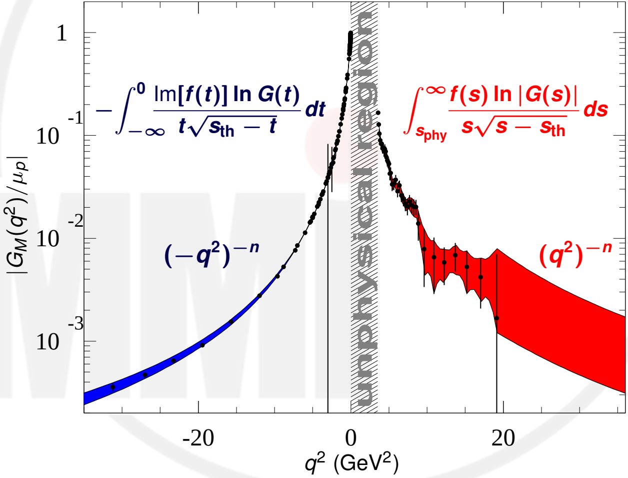
Convergence relation to find the asymptotic power-law behavior of G_M

$$\underbrace{-\int_{-\infty}^{0} \frac{\mathrm{Im}[f(t)] \ln G(t)}{t\sqrt{s_{\mathrm{th}} - t}} dt}_{\text{Space-like data } + (-t)^{-n}} = \int_{s_{\mathrm{th}}}^{\infty} \frac{f(s) \ln |G(s)|}{s\sqrt{s - s_{\mathrm{th}}}} ds \approx \underbrace{\int_{s_{\mathrm{phy}}}^{\infty} \frac{f(s) \ln |G(s)|}{s\sqrt{s - s_{\mathrm{th}}}} ds}_{\text{Time-like data } + s^{-n}}$$

***n* is the only free parameter**

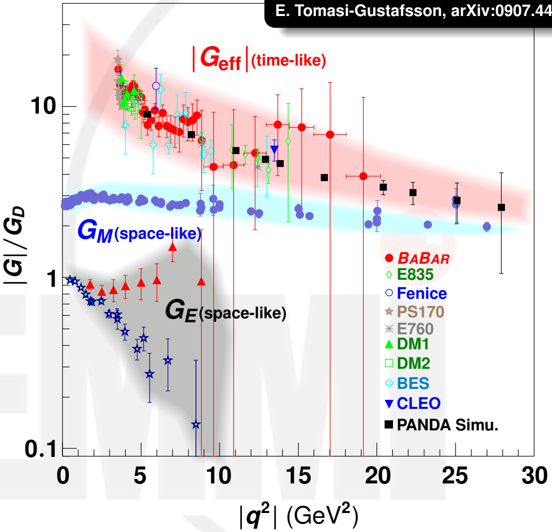
Sum rule: result for G_M

$$G_M(q^2) \underset{|q^2|\to\infty}{\propto} |q^2|^{-(2.27\pm0.36)}$$



Asymptotic behaviors

E. Tomasi-Gustafsson and M. P. Rekalo, PLB504, 291
E. Tomasi-Gustafsson, arXiv:0907.4442



pQCD

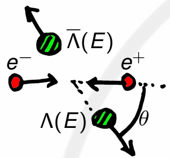
$$G^p_{\rm eff}(q^2) \underset{q^2\to\infty}{\sim} G_M(q^2)$$

Phragmèn Lindelöf

$$\lim_{q^2\to\infty} \frac{G_{\rm eff}(q^2)}{G_M(-q^2)} = 1$$

PHASE AND MODULUS OF $G_{\hat{E}}/G_{\hat{M}}$

Λ form factors



Same definitions, but for labels and Coulomb factor...

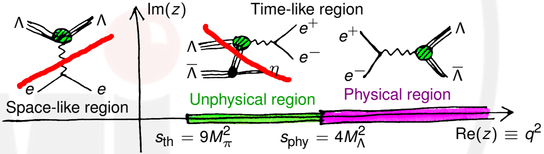
Annihilation cross section

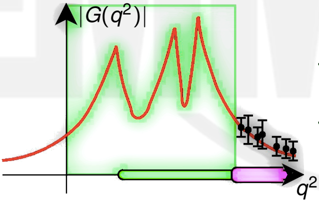
$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2 \beta\, \mathcal{C}}{16E^2}\left[\left(1+\cos^2(\theta)\right)|G_M^{\Lambda}|^2 + \frac{1}{\tau}\sin^2(\theta)\,|G_E^{\Lambda}|^2\right]$$

$$\tau = E^2/M_{\Lambda}^2$$

$$\beta = \sqrt{1-1/\tau}$$

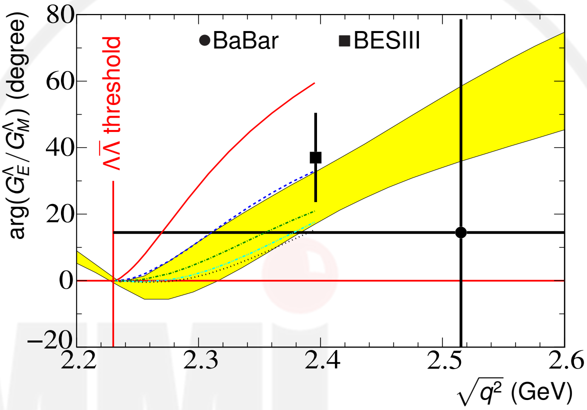
- Same analyticity as for nucleons.
- Difficult to measure in space-like and unphysical regions.
- Relative phase from weak decay.





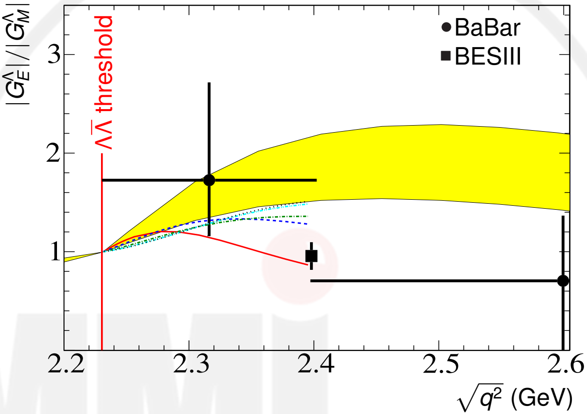
- Same unitarity and intermediate states contributions, but for the isospin.
- Form factors have not vanishing imaginary part above the theoretical threshold.

Phase of $G_E^\Lambda / G_M^\Lambda$



- Theoretical prediction based considering only $\Lambda\bar{\Lambda}$ FSI
  [J. Haidenbauer, U.-G. Meissner, PLB 761 (2016) 456]
- Data from BaBar and BESIII (preliminary)
  [PRD 76 (2007) 092006, arXiv:1903.09421 [hep-ex]]
- "Lambdization" of proton, i.e., proton results with $\sqrt{q^2} \to \sqrt{q^2} + 2(M_\Lambda - M_p)$
  [EPJA32 421 (2007)]

Modulus of $G_E^\Lambda / G_M^\Lambda$



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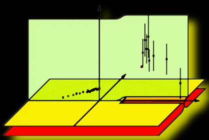
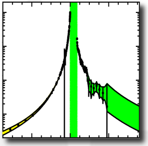
Final considerations



- Space-like zero for G_E
- Time-like phase of G_E/G_M goes to 180^o
- Time-like form factors separation

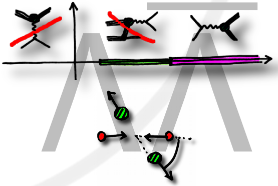


Space-like and time-like "fixed" data on $|G_M^p|$ and analyticity

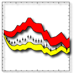
⇓

Confirmation of the pQCD asymptotic behavior

- Relative phase and modulus for the ratio G_E^Λ/G_M^Λ agree with (only) FSI interaction
- It gives information about space-like behavior only if the complex structure is due the intrinsic nature of the baryon-photon vertex
- An asymptotic relative phase of 180^o would imply a space-like zero for G_E^Λ

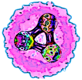


“To do” list



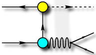
Time-like $|G_E|$-$|G_M|$ separation:

DR and data

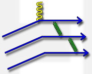


Understanding threshold effect(s):

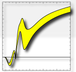
Dispersive analyses: integral equation, sum rule,. . .



Experimental observation in $p\bar{p} \to \pi^0 l^+ l^-$
[PRC75,045205(07)]

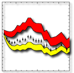


Asymptotic behavior: DR and data for the phase



Zeros $\leftrightarrow$ phases: DR and data

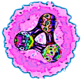
"To do" list



Time-like $|G_E|$-$|G_M|$ separation:

DR and data

BESIII · PANDA · CMD · SND



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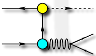
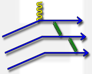


Experimental observation in $p\bar{p} \to \pi^0 l^+ l^-$ [PRC75,045205(07)]

PANDA



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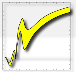
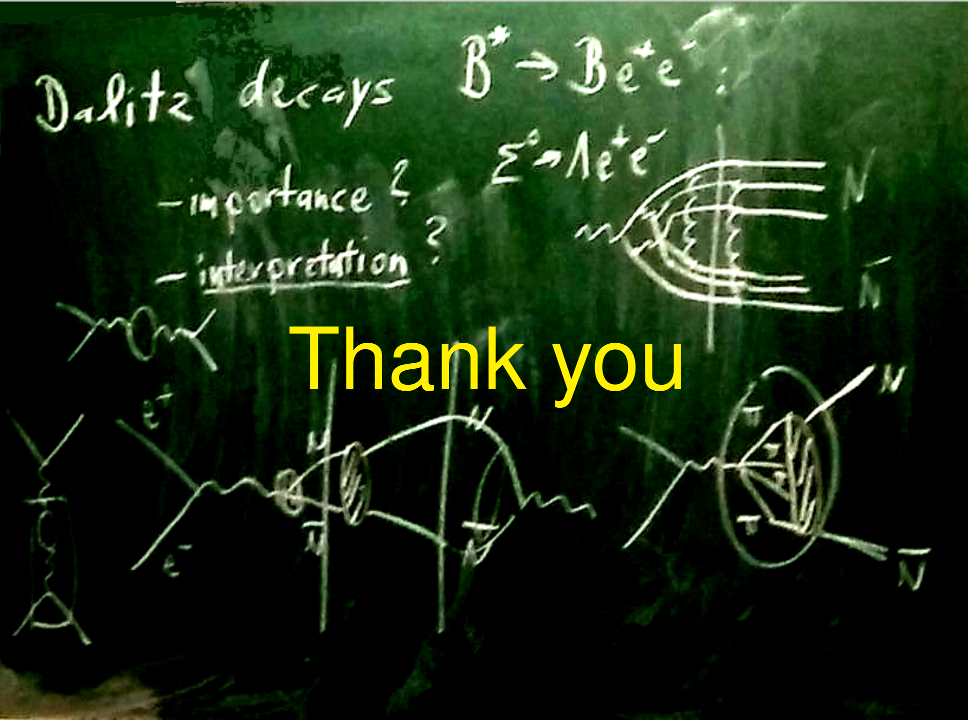
PANDA



Zeros $\leftrightarrow$ phases: DR and data

PANDA · Jefferson Lab



Thank you